\[ in_A(n) = \begin{cases} 1 & \text{if } n \notin A \\ 0 & \text{otherwise} \end{cases} \quad \text{decidable} \]

\text{Semidecidable:}

\[ in_A(n) = \begin{cases} 1 & \text{if } n \notin A \\ \text{runs indefinitely} & \text{if } n \in A \end{cases} \]

If \( A \) is r.e. \( \Rightarrow \) then \( A \) is a semi-decidable
- Have an algorithm that prints all elements in \( A \).

\[ in_A(n) \text{ waits and checks every hour if } n \text{ was printed.} \]

\( \Rightarrow \text{true(1)} \rightarrow n \in A \)
\( \text{runs indefinitely} \rightarrow in_A(n) \)

If \( A \) is semi-decidable then \( A \) is r.e.

26.09

Review: \( \mu \) recursive implementation on TM

Big Pic: \( \mu \) recursive f/n

\[ f \rightarrow TM \]

Proving that every \( \mu \) r.e. fn. is Turing computable.

\[ 0, 0, \pi \rightarrow \bigcirc \quad \text{r.reccursion} \]

\[ \mu m \times p(a^c, m) = f(a^c) \]

While \( \rightarrow \) no condition to stop
We know $\#(a, m)$ is Turing computable.

We wanna produce a TM for computing $\#(a, m)$.

Analyze the problem.

Idea: Let $m = 0$

if $f(a, 0)$ return 0;
else

let $m = 1$

if $f(a, 0)$ return 1;

$m = 0$

while $(f(a, m))$:

$m + 1$;

$\# a \# m \# l$

Start correct way:

$\# a \# a \# a$

Copy $a$

$\# a \# a \# a \# a$

Start here
Big problem with what we had so far:
+ when we had a negative result, it was OK.
+ sometimes, actually produced algorithms, but many of these algorithms were unrealistically long.

Example: \( \sigma(n) = n+1 \).

For security, we have integers: 100 - 200 digit long:

\( 10^{100} \) impossible # of steps.

\( \rightarrow \) we need to separate "theoretical" algorithms from practical algorithms.

\( \rightarrow \) how do you define an algorithm?

\( 1930s \rightarrow \mu, \tau, \sigma \), etc.

\( \rightarrow \) how do you define a feasible algorithm?

No perfect definition exists.

Existing:

- exhaustive search.
- sorting \( O(\log n) \).
A little bit of physics

1 year = 365 days = 3 \times 10^7 seconds

T = \frac{20}{9} billion years

= 2 \times 10^{10} years

= 6 \times 10^{12} seconds

\Delta t < \text{smallest possible time interval}

t = \text{time during which the light goes through the smallest elementary particle}

\text{Heisenberg's uncertainty principle}

\Delta X \Delta p \geq \frac{\hbar}{2} = \text{Plank's constant}

Q: Next Thursday: Quiz / Test