

CNF = conjunctions of disjunction of literals \leftarrow NOT $(x_1 \vee \bar{x}_2)$

+ Convert to Skolem

- change name of bound vars: !!

$$\forall x A \rightarrow C \Leftrightarrow \exists x (A \rightarrow C)$$

$$\exists x A \rightarrow C \Leftrightarrow \forall x (A \rightarrow C)$$

A : it rains a day x $\forall x \text{ rainy-day}(x) \rightarrow \text{stay home}(y)$

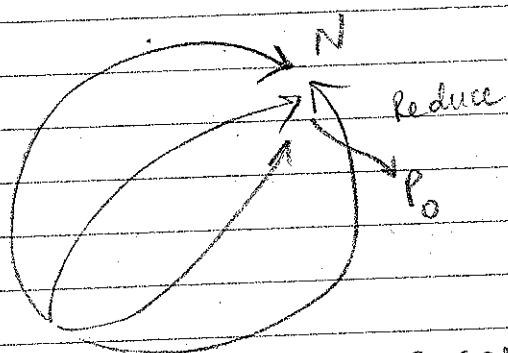
C : I stay home

$$\Leftrightarrow \exists x (\text{rainy-day}(x) \rightarrow \text{stay}(y))$$

$$\exists x \text{ rainy-day}(x) \rightarrow C$$

\Leftrightarrow If there exists a rainy day x then I'll stay home

April 9 \rightarrow



Example 1 = 3 CNF formula

$$+ \text{ CNF : } C_1 \& C_2 \& C_3 \dots$$

$$C_i = a \vee b \vee c \dots$$

} NP
hard

x_i or \bar{x}_i

+ 3-CNF :

$$C_i \leq 3 \text{ literals } a \vee b \vee c$$

$$(\bar{x}_1 \vee x_2 \vee x_3 \vee x_4) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \dots \vee x_3) \wedge \dots$$

Trick : how will the computer do it?

$$r_1 = x_1 \vee x_2$$

$$r_2 = r_1 \vee x_3$$

$$r_3 = r_2 \vee x_4$$

...

...

" $a = b \vee c$ "

a	b	c	f	$\neg f$
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0
0	1	1	0	1

DNF of $\neg f$:

$$(\bar{a} \wedge \bar{b} \wedge \bar{c}) \vee (a \wedge \bar{b} \wedge c) \vee (a \wedge \bar{b} \wedge \bar{c}) \vee (\bar{a} \wedge b \wedge c)$$

\Leftrightarrow CNF of f :

$$f = (a \vee b \vee c) \wedge (\bar{a} \vee b \vee c) \wedge (\bar{a} \vee \bar{b} \vee c) \wedge (a \vee \bar{b} \vee \bar{c})$$

(1)

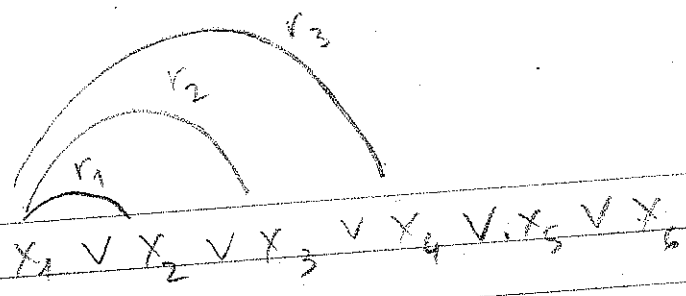
$$\Leftrightarrow (r_1 \vee x_1 \vee x_2)$$

$\begin{matrix} \text{a} & \text{b} & \text{c} \\ \text{r}_1 & = & x_1 \vee x_2 \end{matrix}$

$$\Leftrightarrow (r_1 \vee x_1 \vee x_2) \wedge (r_1 \vee \bar{x}_1 \vee x_2) \wedge (r_1 \vee \bar{x}_1 \vee \bar{x}_2) \wedge (r_2 \vee x_1 \vee x_3)$$

$\begin{matrix} \text{a} & \text{b} & \text{c} \\ \text{r}_2 & = & x_1 \vee x_3 \end{matrix}$

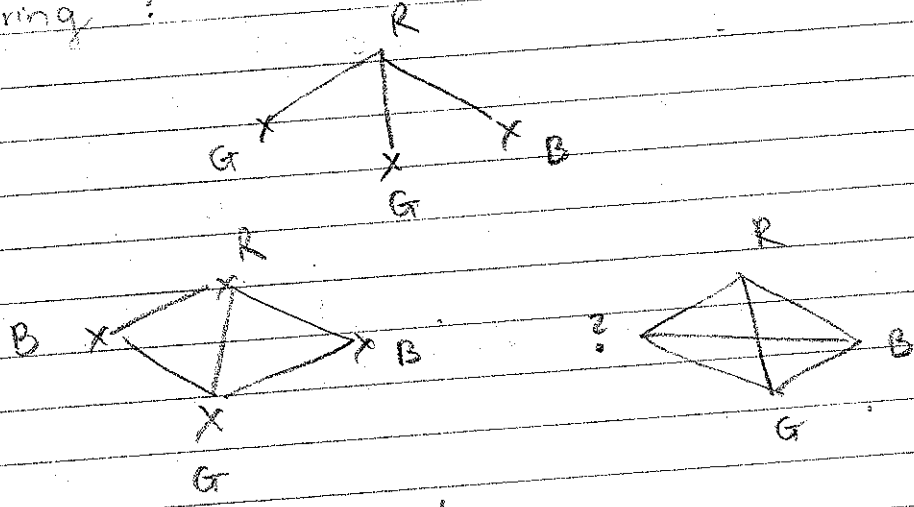
$$(r_2 \vee r_1 \vee x_3) \wedge (r_2 \vee \bar{r}_1 \vee x_3) \wedge (r_2 \vee \bar{r}_1 \vee \bar{x}_3) \wedge (r_2 \vee r_1 \vee x_3)$$



$r_1 \vee X_2 \vee X_3 \vee X_4 \vee \dots$
 $r_2 \vee X_3 \vee X_4 \vee \dots$
 $r_3 \vee X_4 \vee X_5 \vee X_6$

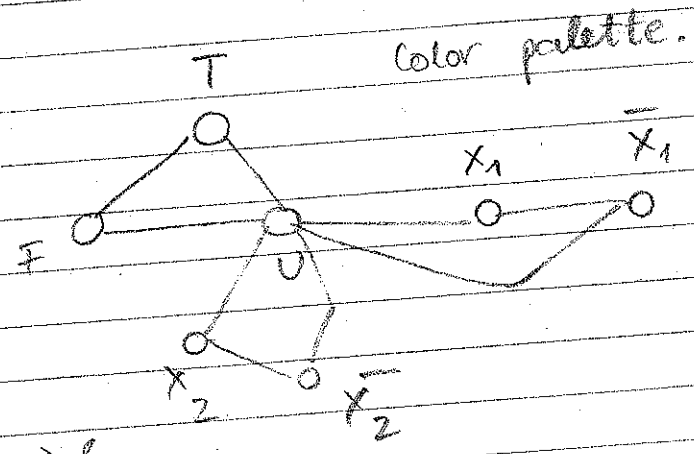
3-CNF we stop.

Coloring :



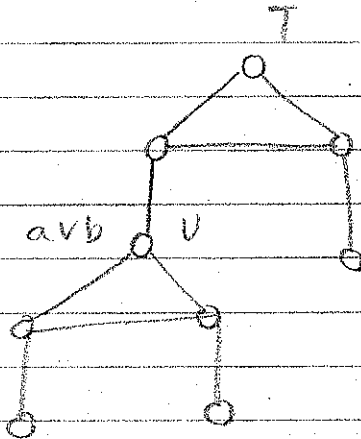
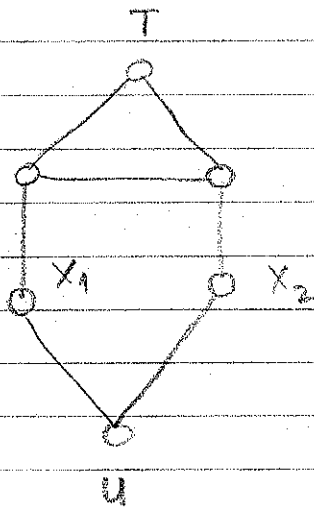
3-coloring is NP-hard

Reduce 3-SAT \rightarrow 3-color



$(a \vee b) \wedge (a \vee b \vee c) \wedge \dots$

OR-gadget

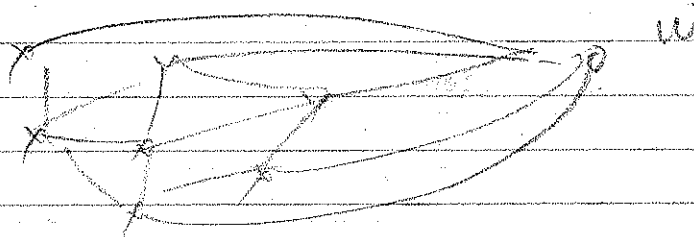


$$(x_1 \vee x_2) \wedge (x_1 \vee x_2 \vee \bar{x}_3)$$



- 1) Take any formula in CNF longer than 3 \rightarrow make 3
- 2) Transform any 3 CNF \rightarrow 3-coloring
- 3) coloring in 5 colors

T F U W



x Subset Sum

S_1, \dots, S_n

S - sum

Find a subset $I \subseteq \{1, \dots, n\}$

$$\sum_{i \in I} S_i = S$$

	x_1	x_2	x_3	x_4	x_5
	1	1	5	10	25

$x_1, x_2, \dots, x_n \in \{0, 1\}$

April 14

MODAL LOGIC

+ possibility, necessity, probability

- Main ops

\square = necessary

\diamond = possible

- Other ops

+ It was once the case that

+ It will once be the case that

+ It ought to be the case that

$$\square P \leftrightarrow \neg \diamond \neg P$$

$$\diamond P \leftrightarrow \neg \square \neg P$$

(\diamond)

(P)

→ It's possible that it'll rain

↔ It's not necessary that it'll ^{not} rain.

$\neg (\square)$

$\neg (P)$

$$f = (a \vee b \vee c) \& (a \vee \bar{b} \vee \bar{c}) \& (\bar{a} \vee b \vee \bar{c}) \\ \& (\bar{a} \vee \bar{b} \vee c).$$



: Extra credit

check if $X \rightarrow Y \Leftrightarrow Y \vee \neg X$

①

$$f = (3.5x_1 - 2.6x_2 \geq 0)$$

$$f = (\bar{x}_1 \wedge \bar{x}_2) \vee (x_1 \wedge \bar{x}_2) \\ \vee (x_1 \wedge x_2)$$

x_1	x_2	f	$\neg f$
0	0	1	0
0	1	0	1
1	0	1	0
1	1	1	0

$$\neg f = \bar{x}_1 \wedge x_2 \rightarrow \text{DNF form for } \neg f$$

prob 14 + Jitn: $a \vee b \vee c \vee d$

In class:

$$\underbrace{a \vee b \vee c \vee d}_{\neg r_1}$$

+ alternative idea:

$$(a \vee b \vee p) \& (\bar{p} \vee c \vee d)$$

+ Resolution

$$(a \vee b \vee p) \wedge (\bar{p} \vee c \vee d) \\ \Leftrightarrow (a \vee b) \vee (c \vee d)$$

NP-hard \equiv can be reduced to the class NP
 NP-complete \equiv NP-hard and belongs to class NP.

+ Subset Sum = Given S_1, \dots, S_n, S
 Find $x_i \in \{0, 1\}$ s.t. $\sum_{i=1}^n x_i S_i = S$

$$F = (\underbrace{x_1}_{C_1} \vee \bar{x}_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \underbrace{x_2}_{C_2} \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \underbrace{x_3}_{C_3})$$

	x_1	x_2	x_3	C_1	C_2	C_3	$S_i \rightarrow n$
corr. to x_1 y_1	1	0	0	1	0	0	100, 100
\bar{x}_1 z_1	1	0	0	0	1	1	100, 011
x_2 y_2		1	0	1	1	0	10, 110
\bar{x}_2 z_2		1	0	0	0	1	10, 001
x_3 y_3			1	0	1	0	1, 010
\bar{x}_3 z_3			1	1	0	0	1, 100
Auxiliary	g_1			1	0	0	100
	h_1			1	0	0	100
	g_2				1	0	10
	h_2				1	0	10
	g_3					1	1
	h_3					1	1
$\rightarrow S$	1	1	1	3	3	3	

- ① If F is satisfiable \rightarrow
 we can find x_i s.t. $\sum x_i S_i = S$
- ② If there is a combination s.t. $\sum x_i S_i = S$
 then F is satisfiable

we pick x_1, x_3, \bar{x}_2 to satisfy F .

$$(x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee x_4) \wedge (\bar{x}_1 \vee \bar{x}_4)$$

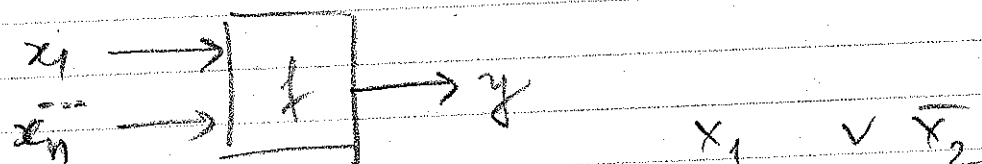
→ No. of clauses = # clauses c_1, c_2, c_3
 →

Ex of NP-hard:

Given: $f(x_1, \dots, x_n)$

$$[x_1, \bar{x}_1], \dots, [x_n, \bar{x}_n]$$

Find: $[y, \bar{y}] = \{ f(x_1, \dots, x_n); x_i \in [x_i, \bar{x}_i], \dots, x_n \in [x_n, \bar{x}_n] \}$



$$(x_1 \vee \bar{x}_2) \quad y = (z_1 + (1 - z_2) - z_1(1 - z_2)) \times (\dots)$$

$$\left(\begin{array}{l} x_i \in \{0, 1\} \\ z_1 = [0, 1] \\ \neg z_2 = 1 - z_2 \end{array} \right) \quad y = 1 \Leftrightarrow F \text{ is satisfiable}$$

$$x \vee y = x + y - x \cdot y$$

$$x \wedge y = x \cdot y$$