Fuzzy Logic.

\[ [0,1] \rightarrow \text{output lies in this} \]

Quiz on CNF, DNF form

Use a general algorithm to find CNF & DNF

1. \( x_1, x_2 \in \{0,1\}, f = "3 \cdot 5 \cdot x_1 - 2 \cdot 6 \cdot x_2 \geq 0" \)

2. "if \((x_1 > x_2)\) then "\((x_1 = x_2)\)"

\[ \text{SAT} \]

\[ \text{reduced to } P_0 \]

If SAT can be reduced to \( P_0 \), then \( P_0 \) is NP-hard.

Example: 3-CNF formula

\[ C_1 \land C_2 \land \ldots \land C_m \]

3-CNF is same as CNF, but

\[ C_i = a \lor b \lor \ldots \lor c \]

\[ \text{NP-hard} \]

\[ \exists x_i \text{ or } \bar{x}_i \]

Now, how we reduce CNF to 3-CNF form? Say, CNF =

\[ \text{Truth table for } \text{"a = b \lor c"} \]

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<tr>
<th>a</th>
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<th>c</th>
<th>f</th>
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Converting to CNF:

\[ f = (a \land b \land c) \lor (a \land \overline{b} \land \overline{c}) \lor (a \land b \land \overline{c}) \lor (a \land \overline{b} \land c) \lor (\overline{a} \land b \land c) \]

* Replaced by clauses,

\[ C_1 \land C_2 \land \ldots \land C_m \]

Then reduced to 3-CNF
\((x_1 \lor x_2 \lor x_3 \lor x_4 \lor x_5 \lor x_6)\) can be broken into:

\((y_1 = x_1 \lor x_2) \lor \)  
\((y_2 = x_1 \lor x_3) \lor \)  
\((y_3 = x_2 \lor x_4) \lor \)  
\((y_4 = x_3 \lor x_5) \lor \)  
\((y_5 = x_4 \lor x_6) \lor \)  
\((y_6 = x_5 \lor x_6) \lor \)  

3-Coloring problem:

- **Diagram:**
  
\(\text{This graph can't be colored with 3 colors.}\)

3-coloring problem is NP-hard:

We know 3-CNF is NP-hard. So, we will reduce it 3-coloring problem.

3 colors are: True, False, Unknown.

To make sure \(x_1, \overline{x}_1\) is not unknown make connection with unknown. This graph can be colored with 3-colors.

**OR-gadget:**

we are sure \(x_1, x_2\) are not \(T\) since connected if we have \(x_1, \overline{x}_1\).

\(\text{we don't know}\)

\(\text{This graph can be}\)
For 3 literals, OR-gadget.

- We should avoid having $a \lor b = \bar{c}$
- $a = \bar{c}$

Example: \((x_1 \lor \bar{x}_2) \lor (x_1 \lor x_2 \lor \bar{x}_3)\)

H.W.

1. Give an example of CNF with > 8 reduce to 3-CNF.
2. Take 3-CNF formula and reduce to 3-coloring.
3. Coloring in 5-colors.

Say 4 colors: T F U W

If we connect with $U \rightarrow W$, then reduced to coloring with T, F, U.

Subset-Sum:

- \(S_1, \ldots, S_n\) - integers no \(S_i > 0\)

- \(S\)-Sum

Find a subset \(I \subseteq \{1, \ldots, n\}\) s.t. \(\sum S_i = S\)

\[x_1, x_2, \ldots, x_n \in \{0, 1\}\]

\[S = \sum_{i=1}^{n} x_i S_i\]

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