

Tuesday
14th April.

Another way to reduce to 3-CNF form :-

$$a \vee b \vee c \vee d.$$

$$\frac{a \vee b \vee p \quad \bar{p} \vee c \vee d}{a \vee b \vee c \vee d} \text{ resolution.}$$

$$= (a \vee b \vee p) \wedge (\bar{p} \vee c \vee d)$$

if p is false $\Rightarrow a \vee b$ is true
if p is true $\Rightarrow c \vee d$ is true.

NP-hard

NP-complete \equiv NP-hard and belongs to the class NP.

Subset sum.

Given : s_1, s_2, \dots, s_n, S find $x_i \in \{0, 1\}$ s.t. $\sum_{i=1}^n x_i s_i = S$

Reduce to 3-CNF.

Say, we have 3-CNF like:

$$\underbrace{(x_1 \vee x_2 \vee x_3)}_{c_1} \wedge \underbrace{(\bar{x}_1 \vee x_2 \vee x_3)}_{c_2} \wedge \underbrace{(\bar{x}_1 \vee \bar{x}_2)}_{c_3} \quad \dots \quad (1)$$

	x_1	x_2	x_3	c_1	c_2	c_3	
(corrs to x_1) $x_1 = y_1$	1	0	0	1	0	0	$s_1 = 100, 100$
$\bar{x}_1 = z_1$	1	0	0	0	1	1	$s_2 = 100, 011$
$x_2 = y_2$	0	1	0	1	1	0	$s_3 = 010, 110$
$\bar{x}_2 = z_2$	0	1	0	0	0	1	$s_4 = 010, 001$
$x_3 = y_3$	0	0	1	0	1	0	$s_5 = 1, 010$
$\bar{x}_3 = z_3$	0	0	1	1	0	0	$s_6 = 1100$
g_1				1	0	0	$s_7 = 100$
h_1				1	0	0	$s_8 = 100$
g_2					1	0	$s_9 = 10$
h_2					1	0	$s_{10} = 10$
g_3						1	$s_{11} = 1$
h_3						1	$s_{12} = 1$

For every clause we have 2 variables (auxiliary variables)

$S = 1111, 3, 3, 3$

1 for columns corr. to variables, 3 for cols corr to clause

For 3-CNF clauses we will have 4 variables/clause (g_i, h_i)

i.e. $S = 1111, 4, 4, 4$

