Tuesday
14th April.

Another way to reduce to 3-CNF form :-

$a \vee b \vee c \vee d$.

$= (a \vee b \vee p) \,\&\, (\bar{p} \vee c \vee d)$

$$\frac{a \vee b \vee p \quad \bar{p} \vee c \vee d}{a \vee b \vee c \vee d} \quad \text{resolution.}$$

If p is false $\Rightarrow a \vee b$ is true

if p is true $\Rightarrow c \vee d$ is true.

NP-hard

NP-complete $\equiv$ NP-hard and belongs to the class NP.

Subset sum.

Given : $s_1, s_2 \ldots s_n, S$ find $x_i \in \{0,1\}$ s.t. $\sum_{i=1}^{n} x_i s_i = S$

Reduce to 3-CNF.

Say, we have 3-CNF like:

$$\underbrace{(x_1 \vee x_2 \vee \bar{x}_3)}_{c_1} \,\&\, \underbrace{(\bar{x}_1 \vee x_2 \vee x_3)}_{c_2} \,\&\, \underbrace{(\bar{x}_1 \vee \bar{x}_2)}_{c_3} \quad \cdots (1)$$

		x_1	x_2	x_3	c_1	c_2	c_3	
(corrs to x_1)	$x_1 \equiv y_1$	1	0	0	1	0	0	$s_1 = 100,100$
	$\bar{x}_1 \equiv z_1$	1	0	0	0	1	1	~~$s_2 = 100,011$~~
	$x_2 \equiv y_2$	0	1	0	1	1	0	$s_3 = 010,110$
	~~$\bar{x}_2 = z_2$~~	0	1	0	0	0	1	$s_4 = 010,001$
	$x_3 = y_3$	0	0	1	0	1	0	$s_5 = 1,010$
	$\bar{x}_3 = z_3$	0	0	1	1	0	0	$s_6 = 1100$
	g_1				1	0	0	$s_7 = 100$
	h_1				1	0	0	$s_8 = 100$
	g_2					1	0	$s_9 = 10$
	h_2					1	0	$s_{10} = 10$
	g_3						1	$s_{11} = 1$
	h_3						1	$s_{12} = 1$
	S	1	1	1	3	3	3	

For every clause we have 2 variables. (auxiliary variables)

$\underbrace{1 \;\; 1 \;\; 1}$ 1 for columns corr. to variables

$\underbrace{3 \;\; 3 \;\; 3}$ 3 for colms corr to clause

For 3-CNF clauses we will have 3 auxiliary variables/clause (g_i, h_i, i)

For 4-CNF we have 4 & 4 $\Rightarrow$ 4

i.e. $S = 111\ldots$

Claims → F is ① here

① if F is satisfiable ⟹ we can find x_i s.t. $\sum x_i s_i = S$

② if there is a combination s.t. $\sum x_i s_i = S$ then F is satisfiable.

we pick up.

① is satisfied when c_1 is true → x_1 is true → y_1

~~c_2 is true → x_3 is true~~ → y_3

~~c_3 is true~~ → $\bar{x}_2$ is true. → z_2.

For c_1 column to make $S = 3$, we need to add two 1's (g_1, h_1)

for c_2 " " , we need to add two 1's (g_2, h_2)

For c_3 " " , - - - - (g_3, h_3)

② Let's take $s_1, s_4, s_5, s_7 \ldots s_{12}$ for which their sum is S

now we need to

~~Interval computation:~~ NP hard.

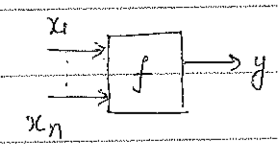
Given : $f(x_1, \ldots, x_n)$ ↗ intervals.

$[\underline{x}_1, \bar{x}_1], \ldots, [\underline{x}_n, \bar{x}_n]$

Find: $[\underline{y}, \bar{y}] = \{ f(x_1, \ldots, x_n) : x_1 \in [\underline{x}_1, \bar{x}_1], \ldots, x_n \in [\underline{x}_n, \bar{x}_n] \}$

range of function

~~height b/w~~



may be height calculation. $[169, 170]$

say, we have CNF formula, $(x_1 \vee \bar{x}_2 \cancel{\vee x_3}) \,\&\, (x_4 \vee x_5 \cancel{\vee \bar{x}_6}) \,\&\, \ldots$

⇓

we can reduce it interval computation.

$x_i \in \{0, 1\}$

$z_i \in [0, 1]$

$y = (z_1 + (1 - z_2) - z_1(1 - z_2)) \cdot (z_4 + z_5 - z_4 \cdot z_5)$

$\neg x_i \to 1 - z_i$

$\bar{y} = 1 \iff F$ is satisfiable

$x \vee y \to x + y - x \cdot y$

$x \,\&\, y \to x \cdot y$