Another way to reduce to 3-CNF form:

\[ a \lor b \lor c \lor d \]

\[ a \lor b \lor p \lor p \lor c \lor d \]

Resolution.

\[ (a \lor b \lor p) \land (p \lor c \lor d) \]

If \( p \) is false \( \Rightarrow \) \( a \lor b \) is true

If \( p \) is true \( \Rightarrow \) \( c \lor d \) is true.

NP-hard

NP-complete = NP-hard and belongs to the class NP.

Subset sum.

Given: \( s_1, s_2, \ldots, s_n, S \) find \( x_i \in \{0,1\} \) s.t. \[ \sum_{i=1}^{n} x_i s_i = S \]

Reduce to 3-CNF.

Say, we have 3-CNF like:

\[ (x_1 \lor x_2 \lor x_3) \land (x_1 \lor x_2 \lor x_3) \land (x_1 \lor x_3) \]

\[ x_1 \times x_2 \times x_3 \]

\[ C_1 \quad C_2 \quad C_3 \]

(\text{Corres to } x_i)

\[ \begin{array}{cccc|cc}
 x_1 & s_1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0
 \hline
 x_2 & s_2 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1
 x_3 & s_3 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1
 x_4 & s_4 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1
 x_5 & s_5 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1
 \end{array} \]

\[ s_i = 100, 010, 100, 001, 010, 100 \]

\[ s_j = 100, 100, 100, 001, 010, 100 \]

For every clause we have 2 variables (auxiliary variables).

For 3-CNF clauses, we still have 5 auxiliary (3 aux).

For columns corresponding to variables, corresponding to clauses, for 4 variables. We have 4 clauses. For 5 variables, we have 5 clauses.
Claims: F is 0 here

1. If F is satisfiable $\Rightarrow$ we can find $x_i$ s.t. $\sum x_i s_i = S$
2. If there is a combination s.t. $\sum x_i s_i = S$ then F is satisfiable.

We pick up.

1. Is satisfied when $c_1$ is true $\Rightarrow x_1$ is true $\Rightarrow y_1$.
   $c_2$ is true $\Rightarrow x_3$ is true $\Rightarrow y_3$.
   $c_3$ is true $\Rightarrow \overline{x}_2$ is true $\Rightarrow z_2$.

For column to make $S = 3$, we need to add two 1's ($y_1, h_1$)

For $c_2$ not, we need to add two 1's ($y_2, h_2$)

For $c_3$ not, we need to add two 1's ($y_3, h_3$)

2. Let's take $s_1, s_4, s_5, s_7... s_{12}$ for which their sum is $S'$

Now we need to

Interval computation: NP hard.

Given: $f(x_1, ..., x_n) \Rightarrow$ intervals:

$[x_1, \overline{x}_1], ..., [x_n, \overline{x}_n]$.

Find: $[y_0, y] = \{ f(x_1, ..., x_n) : x_1 \in [x_1, \overline{x}_1], ..., x_n \in [x_n, \overline{x}_n] \}$

Range of function

\[ \frac{1}{3} \] may be height calculation. [169, 170]

Say we have CNF formula, $(x_1 \lor \overline{x}_3 \lor \overline{x}_5) \land (x_4 \lor \overline{x}_6 \lor \overline{x}_7) \land ...$

We can reduce it (interval computation).

$x_i \in \{0, 1\}$

$z_i \in [0, 1]$

$y = (z_1 + (1 - z_3) - z_4 (1 - z_2)) \cdot (z_4 + z_5 - z_4 z_5)$

$x \lor y \rightarrow x + y - x \cdot y$

$x \land y \rightarrow x \cdot y$