

wel 15

$$a = b \vee c$$

\Leftrightarrow

$$(\bar{a} \wedge b \vee c) \wedge (a \vee \bar{b}c) \wedge (a \vee b \vee \bar{c}) \wedge (a \vee \bar{b} \vee \bar{c})$$

$$(x_1 \vee \bar{x}_2 \vee x_3 \vee x_4) \wedge (\bar{x}_1 \vee x_2 \vee \bar{x}_3 \vee x_4)$$

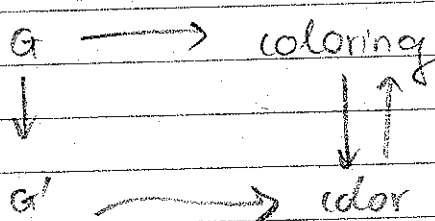
$$\Leftrightarrow (r_1 = x_1 \vee \bar{x}_2) \wedge (r_1 \vee x_3 \vee x_4) \wedge$$

$$(r_2 = \bar{x}_1 \vee x_2) \wedge (r_2 \vee \bar{x}_3 \vee \bar{x}_4)$$

\Leftrightarrow

CNF

?



Schedule

(+)

April 16,

21,

23,

28, — work on project

30 — test 2

May 5 — presentation

7 — when I'm gone

Final ?

Given $f(x_1, \dots, x_n)$
 $[x_1, \bar{x}_1], \dots, [x_n, \bar{x}_n]$

Find :
 $\{ f(x_1, \dots, x_n); x_1 \in [x_1, \bar{x}_1], \dots, [x_n, \bar{x}_n] \}$
 $[y, \bar{y}]$

$$f(x_1, x_2) = x_1 - x_2 \quad \begin{array}{l} x_1 \in [0, 1] \\ x_2 \in [2, 3] \end{array}$$

$$\Rightarrow f(x_1, x_2) : [-3, -1].$$

Range

$$f(x) = x_1^2 - 2x_1 + 5$$

$$x_1 \in [-1, 3]$$

+ find values where $\frac{df}{dx_1} = 0$

+ take values at endpoints

$$2x_1 - 2 = 0$$

$$\text{extreme } x_1 = 1 \rightarrow f(x_1) = 4 \Rightarrow [4, 8]$$

$$\text{end points } \left\{ \begin{array}{l} x_1 = -1 \\ x_1 = 3 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} f(x_1) = 8 \\ f(x_1) = 8 \end{array} \right.$$

Interval computation is NP-hard.

Proof: reduce 3-CNF to it.

$$(x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2)$$

$$x_i \in \{0, 1\} \rightarrow x_i \in [0, 1]$$

$$\neg p \rightarrow 1 - \text{exp.}$$

$$a \vee b \rightarrow a, b$$

$$\neg(a \wedge b) \rightarrow 1 - (1-a)(1-b) \neq$$

$$\begin{array}{ll} x_1 \rightarrow X_1 & \bar{x}_1 \rightarrow 1 - X_1 \\ x_2 \rightarrow X_2 & \bar{x}_2 \rightarrow 1 - X_2 \\ x_3 \rightarrow X_3 & \bar{x}_3 \rightarrow 1 - X_3 \end{array}$$

$$\neg(\bar{x}_1 \wedge x_2 \wedge \bar{x}_3) \wedge (\neg x_1 \vee x_2)$$

$$\begin{array}{ll} \bar{x}_1 \wedge x_2 \wedge \bar{x}_3 & \rightarrow (1 - X_1) \cdot X_2 \cdot (1 - X_3) \\ \neg(\bar{x}_1 \wedge x_2 \wedge \bar{x}_3) & \rightarrow 1 - (1 - X_1) \cdot X_2 \cdot (1 - X_3) \\ \neg(x_1 \wedge \bar{x}_2) & \rightarrow 1 - X_1 \cdot (1 - X_2) \end{array}$$

$$F = (1 - (1 - X_1) \cdot X_2 \cdot (1 - X_3)) \cdot (1 - X_1 \cdot (1 - X_2))$$

$\bar{y} \leq 1$: upper-bound.

$$F = (\bar{v}_1 \vee \bar{v}_2) \wedge (\bar{v}_1 \vee v_2) \wedge (\bar{v}_1 \vee \bar{v}_2)$$

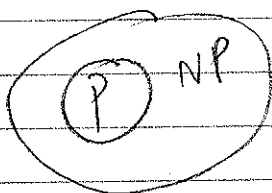
$$F = [1 - (1 - X_1)(1 - (1 - X_2))] \cdot [1 - (1 - (1 - X_1)) \cdot (1 - X_2)] \cdot [1 - (1 - (1 - X_1))(1 - (1 - X_2))] \cdot$$

$$a \vee b = \neg(\bar{a} \wedge \bar{b}) = 1 - (1 - a)(1 - b)$$

$$a \wedge b = a \cdot b$$

$$\begin{aligned} a \wedge b \wedge c &= \neg(\bar{a} \wedge \bar{b} \wedge \bar{c}) = \\ &= 1 - (1 - a)(1 - b)(1 - c) \end{aligned}$$

$$\begin{aligned} a \vee b &= \neg(\neg(a \vee b)) \\ &= \neg(\bar{a} \wedge \bar{b}) \end{aligned}$$



→ beyond NP

→ inside NP

→ NP hard

what do we do?

+ Optimization:
 $f(x_{max})$

$$\forall y (f(y) \leq f(x_{max}))$$

(NP)

Given: x

Find: y s.t. $C(x, y)$
 $\exists y C(x, y)$. (P)

Optim:

given: x

find: y st for every z $C(x, y, z)$.

$$\exists y \forall z C(x, y, z)$$

$$\exists y \forall u \exists v W(y, u, v) \quad \Sigma_3 P$$

$$\forall y \exists u \forall v W(y, u, v) \quad \Pi_3 P$$

$$\Sigma_1 P \subseteq \Sigma_2 P$$

$$\exists y C(x, y) \quad \exists y \forall z C(x, y)$$

← adding fake quantifiers (z)

IC \rightarrow SAT : If you have an "Oracle" for solving SAT, then we can solve IC in polytime.

SAT - an oracle
 IC $\in P$
 NP $\equiv P^{NP}$