l 15

$a = b \vee c$

$\Leftrightarrow$

$(\bar{a} \vee b \vee c) \,\&\, (a \vee \bar{b}c) \,\&\, (a \vee b \vee \bar{c}) \,\&\, (a \vee \bar{b} \vee \bar{c})$

$(X_1 \vee \bar{X}_2 \vee X_3 \vee X_4) \,\&\, (\bar{X}_1 \vee X_2 \vee \bar{X}_3 \vee \bar{X}_4)$

$\Leftrightarrow$ $(r_1 = X_1 \vee \bar{X}_2) \,\&\, (r_1 \vee X_3 \vee X_4) \,\&$

$(r_2 = \bar{X}_1 \vee X_2) \,\&\, (r_2 \vee \bar{X}_3 \vee \bar{X}_4)$

$\Leftarrow$ CNF

$$\begin{tikzcd} G \arrow[r, "?"] \arrow[d] & \text{coloring} \arrow[d, shift right] \\ G' \arrow[r] & \text{color} \arrow[u, shift right] \end{tikzcd}$$

Schedule

+ April 16,
  21,
  23,
  28, — work on project
  30 — test 2

May 5 — presentation
  7 — when I'm gone

Final ?

Given $f(x_1, \dots, x_n)$

$[\underline{x}_1, \overline{x}_1], \dots, [\underline{x}_n, \overline{x}_n]$

Find :

$\{ f(x_1, \dots x_n) ; x_1 \in [\underline{x}_1, \overline{x}_1], \dots, [\underline{x}_n, \overline{x}_n] \}$

$[\underline{y}, \overline{y}]$..

$f(x_1, x_2) = x_1 - x_2 \qquad x_1 \in [0, 1]$

$\qquad x_2 \in [2, 3]$

$\Leftrightarrow f(x_1, x_2) : [-3, -1].$

Range

$f(x) = x_1^2 - 2x_1 + 5$

$x_1 \in [-1, 3]$

+ find values where $\frac{df}{dx_1} = 0$

+ take values at endpoints

$2x_1 - 2 = 0$

extreme $x_1 = 1 \rightarrow f(x_1) = 4 \Rightarrow [4, 8]$

end points $\left\{ \begin{array}{l} x_1 = -1 \\ x_1 = 3 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} f(x_1) = 8 \\ f(x_1) = 8 \end{array} \right.$

Interval computation is NP-hard.

Proof: reduce 3-CNF to it.

$(x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x}_1 \vee x_2)$

$x_i \in \{0, 1\} \longrightarrow x_1 \in [0, 1]$

$\neg p \longrightarrow 1 - \text{exp.}$

$a \wedge b \longrightarrow a \cdot b$

$a \vee b \longrightarrow 1 - (1-a)(1-b)$

$$x_1 \to X_1 \qquad \bar{x}_1 \to 1 - X_1$$
$$x_2 \to X_2 \qquad \bar{x}_2 \to 1 - X_2$$
$$x_3 \to X_3 \qquad \bar{x}_3 \to 1 - X_3$$

$$\neg(\bar{X}_1 \wedge X_2 \wedge \bar{X}_3) \wedge (\neg X_1 \vee \bar{X}_2)$$

$$\bar{X}_1 \wedge X_2 \wedge \bar{X}_3 \to (1-X_1).X_2(1-X_3)$$
$$\neg(\bar{X}_1 \wedge X_2 \wedge \bar{X}_3) \to 1-(1-X_1).X_2.(1-X_3)$$
$$\neg(X_1 \wedge \bar{X}_2) \to 1 - X_1.(1-X_2)$$

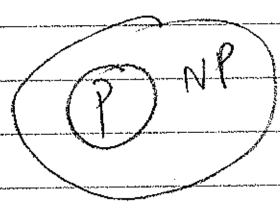
$$F = (1-(1-X_1).X_2.(1-X_3)).(1-X_1.(1-X_2)).$$
$\bar{y} \leq 1$: upper-bound.

$$F = (v_1 \vee \bar{v}_2) \wedge (\bar{v}_1 \vee v_2) \wedge (\bar{v}_1 \vee \bar{v}_2)$$

$$F = [1-(1-X_1)(1-(1-X_2))] * [1-(1-(1-X_1)).(1-X_2)] * [1-(1-(1-X_1))(1-(1-X_2))] *$$

$$a \vee b = \neg(\bar{a} \wedge \bar{b}) = 1-(1-a)(1-b).$$
$$a \wedge b = a.b$$
$$a \wedge b \wedge c = \neg(\bar{a} \wedge \bar{b} \wedge \bar{c}) = 1-(1-a)(1-b)(1-c)$$

$$a \vee b = \neg(\neg(a \vee b)) = \neg(\bar{a} \wedge \bar{b})$$



→ beyond NP
→ inside NP
→ NP hard
what do we do?

+ Optimization :

$f(X_{max})$

$\forall y \ (f(y) \leq f(X_{max}))$

NP Given: x

Find : y s.t $C(x, y)$

$\exists y \ C(x, y)$: P

Optim: given: X

Find : y s.t for every z $C(x, y, z)$.

$$\exists y \ \forall z \ C(x, y, z).$$

$\exists y \ \forall u \ \exists v \ W(y, u, v)$ $\qquad$ $\forall y \ \exists u \ \forall v \ W(y, u, v)$

$\Sigma_3 P$ $\qquad$ $\Pi_3 P$

$\Sigma_1 P \subseteq \Sigma_2 P$ $\leftarrow$ adding fake quantifiers (z)

$\exists y \ C(x, y)$ $\quad$ $\exists y \ \forall z \ C(x, y)$.

IC $\longrightarrow$ SAT : If you have an "Oracle" for solving SAT, then we can solve IC in polytime

Interval comp

SAT - an oracle

$IC \in P^{SAT}$

$NP \equiv P^{NP}$