Thursday.
16th April.

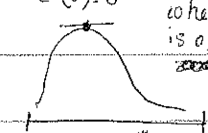
Interval computation :— (IC)

1. $f(x_1, x_2) = X_1 - X_2 \qquad x_1 \in [0,1] \qquad x_2 \in [2,3]$

largest ~~smallest~~ $\bar{y} = \bar{x}_1 - \underline{x}_2 = 1 - 2 = -1 \qquad \therefore$ Range of $y = [-3, -1]$

Smallest $\underline{y} = \underline{x}_1 - \bar{x}_2 = 0 - 3 = -3$

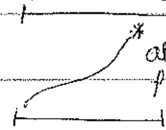
2. $f(x_1) = x_1^2 - 2x_1 + 5 \qquad x_1 \in [-1, 3]$



when derivative is 0, fn value is ~~min~~ max

To find max $f(x_1)$

* find values where $\frac{df}{dx_1} = 0$ ←
* " " at end points. ←



at end point fn is max

$\frac{df}{dx_1} = 2x_1 - 2 = 0 \qquad \therefore x_1 = 1 \qquad f(x_1) = 4 \longrightarrow$ min

at end points, $x_1 = -1 \qquad f(x_1) = 8 \longrightarrow$ max

$x_1 = 3 \qquad f(x_1) = 8$

$\therefore$ Range of $f(x_1) = [4, 8]$.

How to prove: IC is NP hard.

Reduce 3-CNF to IC

Let's start with :- $(v_1 \vee \bar{v}_2 \vee v_3)$ & $(\bar{v}_1 \vee v_2) = \neg(\bar{v}_1 \wedge v_2 \wedge \bar{v}_3)$ &
foreach variable, taking a real-life variable. $\neg(v_1 \wedge \bar{v}_2)$

set consisting of 0,1

$v_i \in \{0, 1\} \longrightarrow X_i \in [0, 1]$

$\neg P. \longrightarrow 1 -$ expr.

$a \& b \rightarrow a.b$

$a \vee b \rightarrow 1 - (1-a).(1-b)$

$v_1 = x_1$ — Now, the equation :—

$v_2 = x_2 \qquad \neg(\bar{v}_1 \wedge v_2 \wedge \bar{v}_3)$

$v_3 = x_3 \qquad = 1 - ((1-x_1).(x_2.(1-x_3)))$

$\bar{v}_1 = (1 - x_1) \qquad \neg(v_1 \wedge \bar{v}_2)$

$\bar{v}_2 = (1 - x_2) \qquad = 1 - (x_1.(1-x_2))$

$\bar{v}_3 = (1 - x_3) \qquad \therefore f = [\underbrace{(1 - (1-x_1).x_2(1-x_3))}_{1}.\underbrace{(1 - x_1.(1-x_2))}_{1}]$

$x_i \in [0,1]$

$1 - x_i \in [0,1]$

Product $x_i \cdot x_j \in [0,1]$ (for f we have only these two operations)

Thus, $f \in [0,1]$

$\therefore \bar{y} \leq 1$ — upper bound of f

$y = 1$ The only way to have this, is when we have one of literals is true.

$\boxed{\begin{array}{l} \rightarrow \text{beyond NP} \\ \rightarrow \text{inside P} \\ \rightarrow \text{NP-hard, what do we do?} \end{array}}$

Beyond NP:- **Optimization.** (function attains max)

$f(X_{max})$ $\quad \forall y\, (f(y) \leq f(X_{max}))$

Given : x

Find : y s.t. for every z $C(x,y,z)$

means, $\exists y \forall z\, (f(y) \geq f(z))$

$\therefore$ described as $\Sigma_2 P$

NP : Given x

Find: y s.t. $C(x,y)$

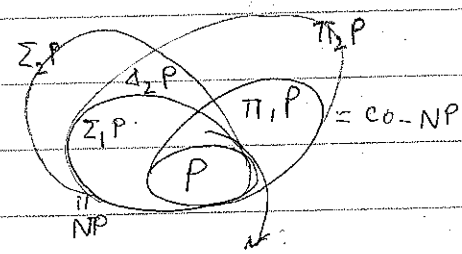
$\exists y\, C(x,y) \equiv \Sigma_1 P$ — P

Polynomial hierarchy

1 for one quantifier

Σ since first " $\exists$

Π for " " $\forall$



~~$\Delta_1 P = \Sigma_1 P \cap \Pi_1 P$~~

Frog-diagram

$\Sigma_1 P \subseteq \Sigma_2 P$

becoz we can write

$\exists y . C(x,y) \subseteq \underbrace{\exists y \forall z\, C(x,y)}_{\Sigma_2 P}$

and also $\subseteq \underbrace{\forall z \exists y\, C(x,y)}_{\Pi_2 P}$

Q3 : what is $\Sigma_3 P$?

where $\Sigma_3 P$ belongs to polynomial hierarchy?

$IC \rightarrow SAT$

means,
If we have an "ORACLE" for solving SAT then we can solve IC in poly time.

$f(x_1, \ldots x_n) \longrightarrow \bar{y}$

$(S_{123} \ldots) \longrightarrow$

we are not counting steps, gives solution.

$IC \in P^{SAT}$ — an oracle

$NP \equiv P^{NP}$

If SAT can be solved with ORACLE then IC can be solved in polytime w.r.t. SAT (oracle)

ORCLE gives ans in 1 step.

$\Pi_2 P = \Pi_1 P$ means we can solve $\Pi_2 P$, if we solved $\Pi_1 P$.

Qs. whether, $NP^A \stackrel{?}{=} P^A$ where A is ORACLE

Nobody can prove NP=P by diagonization, which is a ORACLE.