

ail 21

Problem: polynomial time
may still mean too long

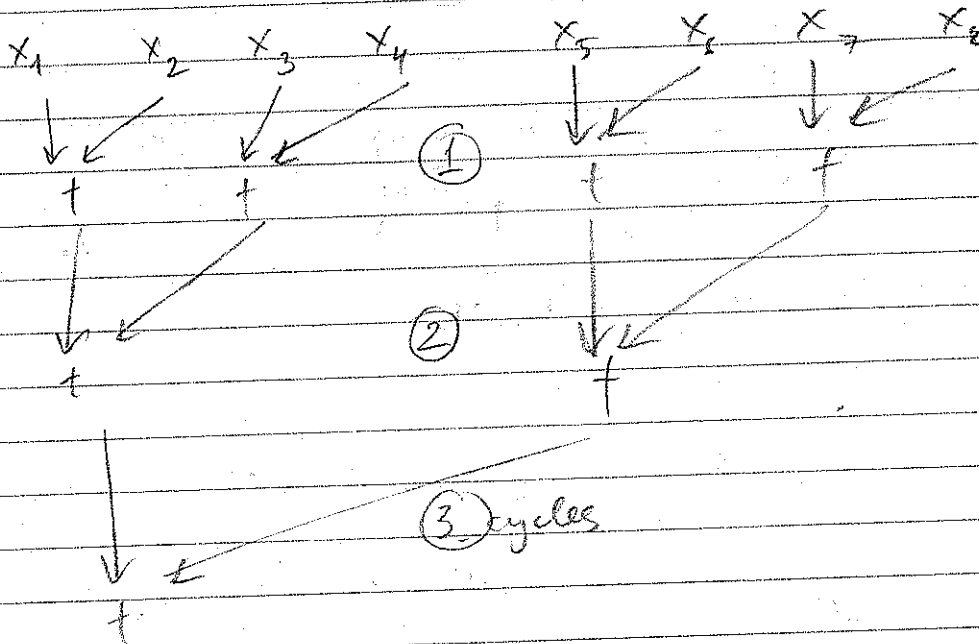
+ Solution: distribution, parallelize,

$$x_1 + x_2 + \dots + x_n$$

If we have 2 processors:

$$(x_1 + x_2 + \dots + x_{n/2}) + (x_{n/2} + \dots + x_n)$$

\downarrow \downarrow
 P_1 P_2



$n = 4$ $t = 2$ $p = 2$

...

$n = 8$ $t = 3$ $p = 4$

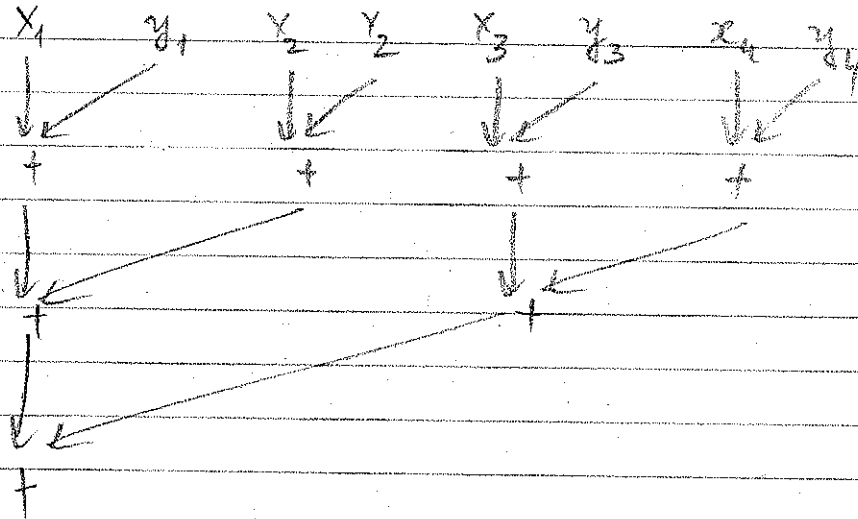
$n = 2^k$ $t = \log_2 n$ $p = \frac{n}{2}$
 $k =$

Dot product:

$$X_1, X_2, \dots, X_n$$

$$Y_1, Y_2, \dots, Y_n$$

$$X \cdot Y = X_1 \cdot Y_1 + X_2 \cdot Y_2 + \dots + X_n \cdot Y_n$$



$$X_1 + \dots + X_n$$

$$p = \frac{n}{2}, \quad t = \log_2 n$$

$$X_1 \cdot Y_1 + X_2 \cdot Y_2 + \dots + X_n \cdot Y_n$$

$$t = 1 + \log_2 m, \quad p = m$$

$$C_{ij} = a_{i1} \cdot b_{1j} + a_{i2} \cdot b_{2j} + \dots + a_{im} \cdot b_{mj}$$

For each C_{ij} , we need $\log_2 m + 1$ time computing all elements of C_{ij} is

$$t = \log_2 m + 1$$

$$p = m^3 \text{ processor.}$$

$$= m^2 \times m$$

m^2
elements

C_{ij} : 1 element

size of input $n = 2m^2$

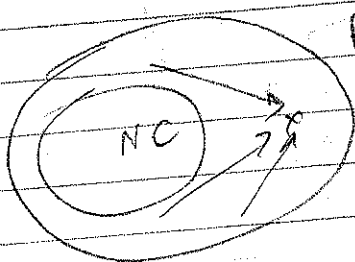
$$\begin{pmatrix} a_{11} & \dots & a_{1m} \\ a_{21} & \dots & a_{2m} \\ a_{31} & \dots & a_{3m} \\ \dots & \dots & \dots \end{pmatrix} \times \begin{pmatrix} b_{11} & \dots & b_{1m} \\ b_{21} & \dots & b_{2m} \\ b_{31} & \dots & b_{3m} \\ \dots & \dots & \dots \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \\ & & \end{pmatrix}$$

Def: A problem is parallelizable if we can solve it in polylog time on polynomial # of processors.
 $t \leq P(\log n)$

NC Nick's class

Nickel = devel iron
 German.

If we have p -processors t -cycles \rightarrow time = $t \times p$
 sequentialized
 $\text{time} \leq P_1(n)$

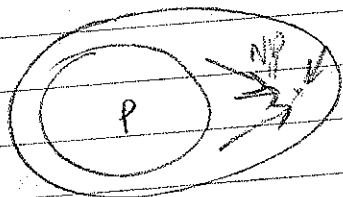


NC \subseteq P

$P \stackrel{?}{=} NC$

Unknown.

P-complete



$NP = P?$

NP-complete.

Exam : P-complete.

Problem : Linear

P-complete

program

Find solutions to systems of linear inequalities.

rice	meat	apples	bread.	milk
X_1	X_2	X_3	X_4	X_5
kg				

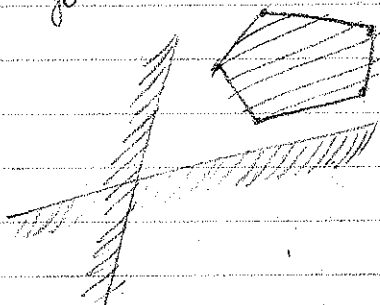
+ calories : $2,000 X_1 + 4,000 X_2 + 100 X_3 + 1000 X_4 + 150 X_5 \geq 2,500$

- proteins : $0. X_1 + 1. X_2 + 0. X_3 + 0.1 X_4 + 0.05 X_5 \geq 0.03$

$0.1 X_1 + 0. X_2 + 1. X_3 + 0.1 X_4 + 0.01 X_4 \geq 0.01$

Price : $1.7 X_1 + 50 X_2 + 2 X_3 + 2 X_4 + 0.75 X_5 \leq 4.0$

How do you solve?



Khachiyan 1978

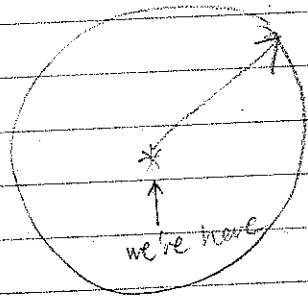
Linear Programming
 $LP \subseteq P$

N. Karmarkar 1984 :
invented a practical poly time algorithm.

Theorem : Linear programming is P-complete.

Ideal world, when we only take into account
comp. time $t \sim \log n$

ii: practice ???



$$R = c \cdot T_{par}$$

$$V_{proc} \leq \frac{V}{\Delta V} = \frac{\frac{4}{3}\pi R^3}{\Delta V} = T_{par}^3$$

$$T_{seq} \leq V_{proc} \cdot T_{par} \leq c \cdot T_{par}^3 \cdot T_{par}$$

$$T_{par}^4 \geq c \cdot T_{seq}$$

We have limitations

1) $v \leq c$

2) $V = \frac{4}{3}\pi R^3$

Euclidean

Geometry



real space-time
is curved.