\[ PA \]

relative to an oracle classes

\[ \text{NPA} \]

\[ \text{NP} \subseteq \text{P}^{\text{SAT}} \]

\[ \text{P} \neq \text{NP} \quad \text{PA} \neq \text{NPA} \]

There are examples:

1) \( A_1 : p^A \neq \text{NP}^A \)

2) \( A_2 : p^A = \text{NP}^A \)

\( \text{co-Simple} \quad A_2 = \text{PSPACE} \)

\( \Sigma_2^P \quad \exists x \forall x \exists y \exists x = \{x, y, z \} \)

\( \Pi_2^P \quad \forall x \exists y \exists x = \{x, y, z \} \)

\( \Sigma_5^P \quad \exists x \forall x \exists y \exists z \forall x \exists y \exists z = \{x, y, z, w \} \)

\[ \exists x, \forall x \exists x, \forall x \exists x \quad \text{win}(\ldots) \]

\[ \exists x, \text{win}(\ldots) \]

\[ \exists x, \forall x \exists x, \text{win}(\ldots) \]

\[ \exists x, \forall x \exists x, \ldots \]

\# of quant. \leq \text{P}(n)

\[ \text{PSPACE} = \text{P}^\text{poly} \quad \text{time}. \]

\[ \text{PSPACE} = \exists x_i, \forall x_{i+1} \quad \text{poly} \quad \# \text{ of quant.} \]

\[ \text{NP} = \bigcup \{ L \mid \exists C \text{ poly}, \forall x, y \in \{0,1\}^* \quad (x,y) \in L \} \]
\[ P(x) \equiv \exists y \in \Sigma^* (x, y) \in \Sigma_1 \text{P} \]

\[ P \equiv \Sigma_0 \text{P} \]
\[ \Sigma_0 \text{P} \subset \Sigma_1 \text{P} \]
\[ \Sigma_1 \text{P} \subset \Pi_2 \text{P} \]
\[ \Pi_2 \text{P} \]

\[ \Delta_1 \text{P} = \Sigma_1 \text{P} \cap \Pi_1 \text{P} \]
\[ \Delta_2 \text{P} = \Sigma_2 \text{P} \cap \Pi_2 \text{P} \]

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**Kolmogorov Complexity of a String**

\[ K(x) = \min \{ \text{len}(p) : \text{program } p \text{ prints } x \} \]

If \[ K(x) \leq \text{len}(x) \], \( x \) is random.

If \[ K(x) < \text{len}(x) \], \( x \) is not random.

\[ K(x) \] is not computable.
two thousand seventeen (3)

Smallest integer that cannot be represented by fewer than twenty words

Proof by contradiction

Assume \( k(w) \) is computable

Let \( L \) be the length of the program that computes \( k(x) \).

\[
\zeta \left\{ \begin{array}{l}
\text{int } \ i = 0; \\
\text{while } (k(i)) < 2 \land L+100 \\
\quad i++;
\end{array} \right.
\]

It prints smallest integer \( x \) for which

\( k(x) > 2 \land L+100 \)

\( L(z) < L+100 \)

Test 3.

3- coloring
Probabilistic algorithm: Monte Carlo algorithm.

we want to find

$$\int_{a_1}^{b_1} \cdots \int_{a_n}^{b_n} f(x_1, \ldots, x_n) \, dx_1 \cdots dx_n$$

Algorithm: Pick $x_i^{(k)} \sim U[a_i, b_i], 1 \leq i \leq n$

$$I = \frac{1}{K} \sum_{k=1}^{K} f(x_1^{(k)}, \ldots, x_n^{(k)})$$

Probability algorithm.

Given: programs $p(x)$ and $p_0(x)$.

Wanted: check whether $\forall x (p(x) = p_0(x))$

Algorithm: Select a random number $\alpha \sim U[a, b]$ and check whether
\[ P(x) \equiv P_0(x) \]
\[ \text{if } f 
eq 0 \]
\[ \forall x (f(x) = f_0(x)) \]
\[ \text{if } = \]
\[ \forall x (f(x) = f_0(x)) \]

**Lobachevsky space**

\[ V(\tau) \sim e^{\tau^2} \]

Processor size \( \Delta V \)

\[ \frac{\Delta V}{\Delta \nu} \sim e^{\frac{\tau^2}{\nu}} = Z_n \]

\[ T = \frac{\tau^2}{C} = O(n) \]

\[ \alpha R = n \hbar^2 \]

\[ R = \frac{n \hbar^2}{2} \]

**Pseudo-BH**

\[ \text{Hypothesis: every even element particle is an entrance to an almost BH to another world} \]

\[ P(\nu, \cdots, \nu) \]
P(P(V_1, ..., V_n))

You pick a random bits X_1, ..., X_n and check whether P(X_1, ..., X_n) is true

true

puzzle

false

finish

Launch time machine to implement a very low probability event with prob. p_0 << 1

-Z-CNFS

\[ X_1 + X_2 + X_3 = 1 \]
\[ X_1 - X_2 - X_3 = 0 \]
\[ 2X_1 + X_2 + X_3 = 2 \]

\[ X_1 = 1 - X_2 - X_3 \]
\[ 1 - X_2 - X_3 - X_2 - X_3 = \emptyset \]
\[ 2(1 - X_2 - X_3) + 2X_3 = 2 \]

\[ 2 - 2X_2 - 2X_3 + 2X_2 + X_3 = 2 \]

\[ 2 - X_3 = 2 \]

\[ X_3 = \emptyset \]