Solution to Problem 9

Problem. Design a Turing machine that computes a function \( f(n) \) which is equal:

- to \( n - 2 \) when \( n \geq 2 \) and
- to 0 when \( n = 0 \) or \( n = 1 \);

it is OK to assume that the input \( n \) is given in unary code.

Solution. The following rules take case of the case when \( n \geq 2 \):

- start, \( \rightarrow \) R, moving
- moving, 1 \( \rightarrow \) R
- moving, \( \rightarrow \) erasing1st
- erasing1st, 1 \( \rightarrow \) \( \_ \), L, erasing2nd
- erasing2nd, 1 \( \rightarrow \) \( \_ \), L, back
- back, 1 \( \rightarrow \) L
- back, \( \rightarrow \) halt

For \( n = 0 \), we get a situation when we are in the state erasing1st and we see blank \( \_ \) since we are in the very first cell. To deal with this case, we need to add a rule

- erasing1st, \( \rightarrow \) halt

For \( n = 1 \), we encounter a similar situation when we are in the state erasing2nd and we see blank. So, we need to add one more rule:

- erasing2nd, \( \rightarrow \) halt