Optimization

\[ f(x_1, \ldots, x_n) \quad x_i \in [x_i, x_i^2] \]

\[ y = \min f(x_1, \ldots, x_n) \quad x_i \in [x_i, x_i^2] \]

\[ \bar{y} = \max f(x_1, \ldots, x_n) \quad x_i \in [x_i, x_i^2] \]

- Branch and Bound -

\[ y = x(1-x) \]

\[ x \in [0, 0.2] \]

\[ f'(x) = 1 - 2x \]

\[ f(x) + f'(x) \cdot [-\Delta, \Delta] \]

\[ f(0.1) = 0.1 \cdot 0.9 = 0.09 \]

\[ 1 - 2 [0, 0.2] \quad [f(0), f(0.2)] \]

\[ = 1 - [0, 0.4] \quad [0, 0.16] \to \text{exact range} \]

\[ = [0.6, 1] \]
2) \([0.2, 0.4]\)
   \(1 - 2[0.2, 0.4] = [0.2, 0.6]\)

\([f(0.2), f(0.4)]\)
\([0.16, 0.24]\) + exact range

So maximum cannot be in \([0, 0.16]\) "since \([0.16, 0.24]\)
is larger."

3) \([0.4, 0.6]\)
   \(\bar{X} = 0.5\)
   \(\Delta = 0.1\)

\(1 - 2[0.4, 0.6] = [0.2, 0.3]\)

\(f(0.5) = 0.15\)

\(0.25 + [-0.2, 0.2][-1, 1] = [-0.23, 0.27]\)

4) Now for \([0.6, 0.8]\)
   \([f(0.8), f(0.6)]\)

\(1 - 2[0.6, 0.8] = [0.2, 0.02]\)
\([0.16, 0.34]\)
   // dismissed

5) \([-0.5, 1]\)

\(1 - 2[-0.8, 1] = [-1, -0.6]\)
\([f(0.1), f(0.8)]\)
\([0, 0.16]\)
   // dismissed
Bisect: \([0.4, 0.5], [0.5, 0.6]\)

\[1 - 2[.4, .5] = \]
\[= [0.2, 0.25]\]

\[1 - 2[.5, .6] = \]
\[= [-.2, 0] \]

So the maximum is at 0.5!!

---

Diagram: A horizontal line segment from 0.4 to 0.6 with a label indicating a maximum at 0.5.