How to locate the max?

Given: \( F(x_1, \ldots, x_n) \)

Box \((x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \)

We want location, where the maximum is set which is guaranteed to contain all the values \( y = (x_1, \ldots, x_n) \) where \( \text{max} \) is obtained \( F(x) = \max_{y \in B} F(y) \)

\[ F(x) = x - x^2 \quad x \in [0, 0.8] \quad F'(x) = 1 - 2x \]

(1) Bisect \([0, 0.4] \quad [0.4, 0.8]\]

\[ F'(0, 0.4) = 1 - 2 \cdot 0.4 = -0.2 \]

\[ F(0.4) = 0.24 \]

We know the max can be attained only \( x \in [0.4, 0.8] \)

\( x \in [0.4, 0.6] \)

\[ F'(0.4, 0.6) = -0.2, 0.2 \]

\[ F(0.5) = 0.25 \]

\( 0.25 = \left[ -0.2, 0.2 \right] \cdot [0.1, 0.1] = [0.23, 0.27] \)

Max is attained \( x \in (0.4, 0.6) \)

\( x \in [0.4, 0.8] \)

\( F'(0.4, 0.8) = [-0.6, 0.2] \)

\( x = 0.6 \)

\[ F(0.6) = 0.04 \]

\( 0.04 = [-0.6, 0.2] \cdot [0.2, 0.2] = [0.12, 0.36] \)

\( x \in [0.4, 0.8] \)

\[ F'(0.4, 0.8) = [0.6, -0.2] \]

decreasing monoton, dismiss the interval \( f(0.6) = 0.24 \)

smaller than largest so far
\[ x \in [0.4, 0.5] \]
\[ F'(x) = 1 - 2x \| 0.4, 0.5] = [0, 0.2] \]
only keep 0.5
\[ x \in (0.5, 0.6] \]
\[ F'(x) = 1 - 2x \| 0.5, 0.6] = [-0.2, 0] \]
keep 0.5
max is located at \( x = 0.5 \)

Another example
\[ f(x) = x - 2x^2 \quad x \in [0.2, 1, 0] \]
\[ F'(x) = 1 - 4x \]
\[ x \in (0.2, 0.6] \]
\[ F'(x) = 1 - 4x \| 0.2, 0.6] = [-1, -0.4, 0.2] \]
max so far
\[ x = 0.4 \]
\[ \Delta = 0.2 \]
\[ f(0.4) = 0.4 - 2 \cdot 0.4^2 = 0.08 \]
\[ 0.08 + [-0.38, 0.38] = [-0.3, 0.36] \]
cannot dismiss
\[ x \in (0.6, 1, 0] \]
\[ F'(x) = 1 - 4x \| 0.6, 1, 0] = [-1, -0.4, 0.6, 1, 0] \]
max at \( x = 0.6 \)
\[ f(0.6) = 0.6 - 2 \cdot 0.6^2 = 0.06 - 0.72 \]
\[ = 0.12 \quad \text{dismiss} \ (0.6, 1, 0] \]

\[ x \in (0.4, 0.6] \]
\[ F'(x) = 1 - 4x \| 0.4, 0.6] = [-1, -0.4, 0.4, 0.6] \]
max at \( x = 0.4 \)
\[ f(0.4) = 0.4 - 2 \cdot 0.4^2 = 0.08 \]
dismiss whole thing
\[ x_1 - x_2 = 0 \quad x_1 \in [0, 2] \]
\[ x_2 \in [0, 2] \]
\[ x_1 \cdot x_2 = 1 \]

1. \[ x_1 \rightarrow x_2 = x_1 \]
2. \[ x_2 \rightarrow x_1 = x_2 \]
3. \[ x_1 \rightarrow x_2 = \frac{10}{x_2} \]
4. \[ x_2 \rightarrow x_1 = \frac{10}{x_2} \]

\( R_3 \) \[ \frac{10}{x_2} \leq 5 \quad [0, 2] \]

No intersection, no solution \( \emptyset \)