f(x) = x + 2x^2, \ x \in [0, 0.2] \\
f'(x) = 1 + 4x, \ x = 0.1 \\
\Delta = .01

Exact Range:

<table>
<thead>
<tr>
<th>x = 0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>x = 0.2</td>
<td>.28</td>
</tr>
<tr>
<td>f(x) = 0</td>
<td>X</td>
</tr>
</tbody>
</table>

f(0) = 0 + 2(0)^2 = 0 \\
f(0.2) = 0.2 + 2(0.2)^2 = .28 \\
\Theta = 1 + 4x \\
-1 = \frac{4x}{4} \Rightarrow x = -1 \\
\frac{1}{4} \Rightarrow X = \frac{1}{4} \text{ which is not in range}

Range: [0, .28]

Algorithm 1:

\[ \tilde{y} = f(0.1) = .1 + 2(0.1)^2 = .12 \]

\[ f'(\tilde{x}) = 1 + 4(0.1) = 1 + 0.4 = 1.4 \]

\[ \Delta = | f'(\tilde{x}) | \Delta \]

\[ \Delta = 1.4 \cdot 0.1 \]

Range: \[ \tilde{y} - \Delta, \tilde{y} + \Delta \Rightarrow .12 - .14, .12 + .14 \]

Range: [-.02, .26] \Rightarrow has a closer estimate on lower bound

Algorithm 2:

\[ \Delta = | f(\tilde{x} + \Delta) - \tilde{y} | \]

\[ f(\tilde{x} + \Delta) = f(0.2) = .28, \ \tilde{y} = .12 \]

\[ 1.28 - .12 | = .16 \]

\[ \Delta = \tilde{y} - \Delta, \tilde{y} + \Delta \Rightarrow .12 - .16, .12 + .16 \]

Range: [-.04, .28] \Rightarrow has exact estimate on upper bound
3) \(d(A) = .7\)
\(d(B) = .6\)
\(d(C) = .8\)
\((A \lor B) \& C\)

\[A \lor B = \max(A, B)\]
\[A \lor B = \max(.4, .6)\]
\[A \lor B = .6\]

\[(A \lor B) \& C = \min(A \lor B, C)\]
\[= \min(.6, .8)\]
\[= .6\]

\[d((A \lor B) \& C) = .6\]

4) \(\mu(x) = |1-2-x|, \alpha = 0.6, \alpha = 0.9\)

\[
\begin{align*}
1-|2-x| &= 0.6 \\
1-|2-x| &= 0.7 \\
1+2-x &= 0.4 \\
3-x &= 0.4 \\
1-2+x &= 0.6 \\
-1+x &= 0.6 \\
x &= 2.3 \\
x &= 1.9
\end{align*}
\]

\[
\begin{array}{c|c|c|c}
\mu(0.6) &= [1.6, 2.4] \\
\mu(0.9) &= [1.7, 2.3]
\end{array}
\]
\[ \mu(x) = 1 - 1|x| \]
\[ \mu(y) = 1 - |2 - y| \]
\[ z = x - y, \quad t = x \times y \]

\[ \frac{1}{2} \]
\[ 1 - 1|x| \]
\[ 1 - x = \alpha, \quad 1 + x = 0.2 \]
\[ 1 - \alpha = x, \quad x = \alpha - 1 \]
\[ 0.2 = [-0.8, 0.8] \]
\[ 0.4 = [-0.6, 0.6] \]
\[ 0.8 = [-0.2, -2] \]
\[ 1 = [0, 0] \]
\[ 0.6 = [-0.4, 0.4] \]

\[ z = x - y \]
\[ 0.2 = [-0.8, 0.8] - [1.2, 2.8] = [-3.6, -4] \]
\[ 0.4 = [-0.6, 0.6] - [1.4, 2.6] = [-2.2, -2.8] \]
\[ 0.8 = [-0.2, -2] - [1.8, 2.2] = [-2.6, -1.8] \]
\[ 1 = [0, 0] - [2, 2] = [-2, -2] \]
\[ 0.6 = [-0.4, 0.4] - [1.6, 2.4] = [-2.2, -1.2] \]

\[ t = x \times y \]
\[ 0.2 = [-0.8, 0.8] \times [1.2, 2.8] = [-2.24, 2.24] \]
\[ 0.4 = [-0.6, 0.6] \times [1.4, 2.6] = [-1.56, 1.56] \]
\[ 0.8 = [-0.2, -2] \times [1.8, 2.2] = [-1.44, -1.44] \]
\[ 1 = [0, 0] \times [2, 2] = [0, 0] \]
\[ 0.6 = [-0.4, 0.4] \times [1.6, 2.4] = [-0.96, -0.96] \]
7) \[ x^2 - 5 = 0 \] \[ [0, 4] \]

\[ u = 0 \]
\[ u = 4 \]
\[ f(0) = -5 \]
\[ f(2) = -1 \]
\[ f(4) = 11 \]

\[ f(1) \]
\[ f(2) = -1 \]
\[ f(3) = 4 \]
\[ f(4) = 11 \]

Solution is enclosed in: \[ [2, 2.5] \]

8) public static double sum (double[] x, double h) { 
    double[] ar1 = new double[x.length];
    double[] ar2 = new double[x.length];
    double sum = 0;
    for (int i = 0; i < x.length; i++) {
        for (int j = 0; j < x.length; j++) {
            if (j == i) {
                ar1[j] = x[j] + h;
            } else {
                ar2[j] = x[j] - h;
            }
        }
        sum += (f(ar1) + f(ar2) - 2 * f(x)) / (4 * h);
    }
    return sum;
}
There is no alternative that is guaranteed to be optimal because none meets the following criteria:

$$\mu_i \geq \max \mu_j$$

**Alternatives that can be optimal:**

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$\alpha = 0$</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[10, 15]</td>
<td>10*</td>
<td>12.5</td>
<td>15</td>
</tr>
<tr>
<td>[4, 9]</td>
<td>4</td>
<td>6.5</td>
<td>9</td>
</tr>
<tr>
<td>[7, 20]</td>
<td>7</td>
<td>13.5*</td>
<td>20*</td>
</tr>
</tbody>
</table>

$$\alpha \mu + (1 - \alpha) \bar{\mu}$$

$\alpha = 0 \Rightarrow 1(\mu) = \mu$

$\alpha = 0.5 \Rightarrow 0.5(\mu) + 0.5\bar{\mu}$

$\alpha = 1 \Rightarrow \alpha \mu = \mu$

**Selecting Alternatives:**

- $\alpha = 0$ select [10, 15]
- $\alpha = 0.5$ select [7, 20]
- $\alpha = 1$ select [4, 20]

---

10)

**Optimizing Compiler:**

Implies: $X - X$, optimizing compiler will give $[0, 0]$ as a result in $IC$

**Example:**

$X \in [-1, 1]$

$[-1, 1] - [-1, 1] = [-2, 0]$

$[0, 0]$ is within range of actual result and has a smaller width.
\[
\frac{1}{1 + \frac{b}{a}} \text{, optimizing compiler will transform expression to } \frac{a}{a + b} \text{. For IC this is worse because every variable is used more than once.}
\]

Example:

\[a \in [1, 2] \text{, } b \in [2, 4]\]

\[
\frac{1}{1 + \frac{[2, 4]}{[1, 2]}} = \frac{1}{1 + [1, 4]} \quad [2, 5] = [\frac{1}{5}, \frac{1}{2}] = [0.2, 0.5]
\]

\[
\frac{[1, 2]}{[1, 2] + [2, 4]} \quad [3, 6] = \frac{[\frac{1}{6}, \frac{2}{3}]}{[0.16, 0.66]}
\]

Result of optimizing compiler has larger width.

\[11) \quad [0.40, 0.75] \times [0.497, 1.30] = [0.208, 0.975] \]

\[0.208, 0.52, 0.5975, 0.975 \quad \text{Round down on lower bound}
\]

\[\frac{1}{36}, \frac{1}{38} \quad \text{Round up on upper bound}
\]

\[\frac{1}{36}, \frac{1}{38} \quad [0.30, 0.98] \]