As we mentioned in class, the general idea of processing data with different accuracies $\sigma_i$ is to minimize the sum $\sum \frac{e_i^2}{\sigma_i^2}$, where $e_i$ is the discrepancy of corresponding to the $i$-th measurement. In the case when we simply measure one quantity $a$ and get the result $x_i$, the discrepancy is $e_i = a - x_i$.

As we discussed in class, the sum to-be-minimized can be equivalently described as $\sum w_i \cdot e_i^2$, where we denoted $w_i = \sigma_i^{-2}$. In the contents of Problem 22, this means minimizing the sum

$$\sum w_i \cdot (a - x_i)^2 = w_1 \cdot (a - x_1)^2 + w_2 \cdot (a - x_2)^2 + \ldots + w_n \cdot (a - x_n)^2.$$ 

Differentiating this sum with respect to $a$ and equating the derivative to 0, we conclude that

$$2w_1 \cdot (a - x_1) + 2w_2 \cdot (a - x_2) + \ldots + 2w_n \cdot (a - x_n) = 0.$$ 

We can divide both sides of this equality by 2, getting

$$w_1 \cdot (a - x_1) + w_2 \cdot (a - x_2) + \ldots + w_n \cdot (a - x_n) = 0,$$

i.e.,

$$w_1 \cdot a - w_1 \cdot x_1 + w_2 \cdot a - w_2 \cdot x_2 + \ldots + w_n \cdot a - w_n \cdot x_n = 0.$$

If we keep all terms proportional to $a$ in the left-hand side and move all the other terms to the righthand side, we get

$$a \cdot (w_1 + w_2 + \ldots + w_n) = w_1 \cdot x_1 + w_2 \cdot x_2 + \ldots + w_n \cdot x_n,$$

hence

$$a = \frac{w_1 \cdot x_1 + w_2 \cdot x_2 + \ldots + w_n \cdot x_n}{w_1 + w_2 + \ldots + w_n},$$

i.e., using the notations that we introduced in class, $a = \frac{x}{1}$. 

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