Example of Estimating Kolmogorov Complexity

**Question.** What can we say about the Kolmogorov complexity of the string 011011..., in which the sequence 011 is repeated 5000 times?

**Discussion.** By definition, the Kolmogorov complexity $K(x)$ of a string $x$ is the length $\text{len}(p_0)$ of the shortest program $p_0$ that prints this string.

Kolmogorov complexity is not computable, so we cannot compute $K(x)$, but we **can** find an upper bound for $K(x)$. Indeed, if we have a program $p$ that prints the string $x$, then, by definition, $K(x) \leq \text{len}(p)$.

**Solution.** We can write a simple loop to print this sequence:

```java
for(int i=1, i<5000, i++)
    System.out.print("011");
```

This program $p$ has $\text{len}(p) = 51$ symbols, so we conclude that $K(x) \leq 51$.

**Is this sequence random?** Random means $K(x) \geq \text{len}(x) - C$, for $C \leq 10$. In our case,

$$\text{len}(x) = 3 \cdot 5000 = 15000,$$

so

$$\text{len}(x) - C \geq 15000 - 10 = 14990,$$

while $K(x) \leq 51$. Clearly, $K(x) < \text{len}(x) - C$, so the sequence $x$ is not random.