Solution to Problem 20

**Task.** We have shown that the only maximally individually robust “or”-operation is \( \max(a, b) \). Maximally individually robust means, in this case, that for all possible values \( a, b, a', \) and \( b' \), we must have

\[
|f_\lor(a, b) - f_\lor(a', b')| \leq \max(|a - a'|, |b - b'|).
\]

Provide an example of the values \( a, b, a', \) and \( b' \), showing that the “or”-operation \( a + b - a \cdot b \) is not maximally individually robust. Hint: it is sufficient to consider values 0, 0.5, and 1.

**Solution.** For \( a = b = 0 \) and \( a' = b' = 0.5 \), we have \( f_\lor(a, b) = f_\lor(0, 0) = 0 \) and

\[
f_\lor(a', b') = f_\lor(0.5, 0.5) = 0.5 \cdot 0.5 - 0.5 \cdot 0.5 = 0.25.
\]

So here \( |f_\lor(a, b) - f_\lor(a', b')| = |0 - 0.25| = 0.25 \).

On the other hand, here, \( |a - a'| = |b - b'| = |0 - 0.5| = 0.5 \), so

\[
\max(|a - a'|, |b - b'|) = \max(0.5, 0.5) = 0.5.
\]

Thus, here

\[
|f_\lor(a, b) - f_\lor(a', b')| > \max(|a - a'|, |b - b'|).
\]