How We Humans Fuse Different Types of Uncertainty when Making Decisions

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Introduction

In many practical situations, we need to make a decision.

In many applications, we do not know the exact consequences of each action.

In such situations, we need to make a decision under uncertainty.

In many application areas, uncertainty is small and can be made even smaller by extra measurements.

For example, for a self-driving car, we can accurately measure all the related values and events.

However, there are applications when it is difficult to decrease uncertainty.

One such area is anything related to humans.

Humans make individual decisions based on their perceived value of different alternatives.

A Rational Agent Should Maximize Utility

Suppose that we have found the utilities of \( A_i, A_j, \ldots \) of the alternatives \( A, A' \).

Which of those alternatives should we choose?

By definition of utility, we have:

\[ U(A) = U(A_i) \]

for every alternative \( A \).

And:

\[ U(L) < U(P) \]

if only if \( P \).

We can thus conclude that \( A' \) is preferable to \( A \) if and only if \( \Delta U > 0 \).

In other words, we should always select an alternative with the largest possible value of utility.

How to Estimate Utility of an Action

For each action, we usually know possible outcomes \( S_1, S_2, \ldots \).

We can often estimate the prob. \( p_1, p_2, \ldots \) of these outcomes.

By definition of utility, each situation \( S_i \) is equiv. to a lottery \( L(S_i) \) in which we get:

\[ U_i \text{ with prob. } p_i(S_i) \text{ and } 0 \text{ with prob. } 1-p_i(S_i) \]

Thus, the action is equivalent to a complex lottery in which:

1. we first select one of the situations \( S_i \) with prob. \( p_i(S_i) \).
2. Then, depending on \( S_i \), we get \( A_i \) with prob. \( p(A_i | S_i) = U_i(S_i) \) and \( A' \) with prob. \( 1-U_i(S_i) \).

Reminder:

1. we first select one of the situations \( S_i \) with prob. \( p_i(S_i) \).
2. Then, depending on \( S_i \), we get \( A_i \) with prob. \( p(A_i | S_i) = U_i(S_i) \) and \( A' \) with prob. \( 1-U_i(S_i) \).

The prob. of getting \( A_i \) in this complex lottery is:

\[ P(A_i) = \sum P_i(A_i | S_i) P(S_i) = \sum p_i U_i = P(A_i) \]

In the complex lottery, we get:

\[ A_i \text{ with prob. } p_i(S_i) = U_i(S_i) \]

or:

\[ A_i \text{ with prob. } 1-p_i(S_i) = 1-U_i(S_i) \]

Thus, we should select the action with the largest expected utility \( E_i = \sum p_i U_i \).

The Notion of Utility

Under the above assumption, we can form a natural numerical scale for describing preferences.

Let us select a very bad alternative \( A_0 \) and a very good alternative \( A_1 \).

Then, most other alternatives are better than \( A_0 \) but worse than \( A_1 \).

For every prob. \( p \), we can form a lottery \( L(p) \) in which we get \( A_1 \) with prob. \( p \) and \( A_0 \) with prob. \( 1-p \).

When \( p = 0 \), this lottery simply coincides with the alternative \( A_0 \).

The larger the probability \( p \) of the positive outcome increases, the better the result:

\[ p \rightarrow 1 \text{ implies } L(p) \rightarrow L(1) \]

Finally, for \( p = 1 \), the lottery coincides with the alternative \( A_1 \).

Thus, we can form a continuous scale of alternatives \( A_0 \).

Due to monotonicity, when we increase, we first have \( L(p) \) and then have \( L(p') \).

The threshold value is called the utility of the alternative \( A_0 \).

Decision Theory: A Brief Reminder

The Notion of Utility

Decision Making Under Interval Uncertainty

In real life, we rarely know the exact consequences of each action.

So, for an alternative \( A \), we often only know the bounds on \( U(A) \) and \( U(A') \).

For such an interval case, we need to be able to compare the interval-valued alternative with lotteries \( L(A) \).

As a result of such comparison, we will come up with an utility of this interval.

So, we need to assign, to each interval \([a, b]\), a utility value \( U(a, b) \).

Our model: utility is determined modulo a linear transformation \( u' = u + b \).

Reasonable to require: the equivalent utility does not change with rescaling: for \( u > 0 \) and \( b \),

\[ u_a = u + b \]

For \( u < 0 \),

\[ u_a = u - b \]

Then, we get \( u_a = u + b \) and \( u_a = u - b \).

This formula was first proposed by a future Nobelist Leon Hurwicz.

It is known as the Hurwicz optimism-pessimism criterion.

Is "No Trade Theorem" Really a Paradox

One of the challenges in foundations of finance is the so-called "no trade theorem" paradox:

- if a trader wants to sell a stock, he/she believes that this stock will go down;
- however, another trader is willing to buy it;
- this means that this other expert believes that the stock will go up.

The fact that equally good experts have different beliefs should disqualify the first expert from selling.

Thus, trades should be very rare.

However, in reality, trades are ubiquitous: how can we explain this?

To Practical Applications of Decision Theory

The numerical value of utility depends on the selection of the alternative, \( A_i \).

If we select a different alternative, \( A_i \), then utility changes into \( U_i(A) = U_i(A_i) + A_i \) for some \( A \neq 0 \).

The dependence of utility of money is non-linear.

Utility is proportional to the square root of the amount \( m \) of money:

\[ U(m) = \sqrt{m} \]

If we have an amount \( m \) of money now, then we can place it in a bank and add an interest.

So, we get the new amount \( m' \)

\[ \frac{m'}{m} = (1 + i) \text{ a year.} \]

Thus, the amount \( m' \) in a year is equivalent to the amount \( m' \) now, where \( m' = m' \).

This is called discounting.

Why Prices for Buying and Selling Objects Are Different

Intuitively, we should decide, for ourselves, how much each object is worth to us.

This worth amount should be the largest amount that we should be willing to pay if we were buying this object.

This same amount should be the smallest amount for which we should agree to sell this object.

However, in practice, the buying and selling prices are different.

Our Explanation

The main reason is that people are not clear on the value of each object.

At best, they have a range \([a, b]\) of possible values of this object’s worth.

According to Hurwicz formula, when we buy, we gain the value \( u_a = a + b \).

On the other hand, if we already own this object and we sell it, then our loss is \( u_b = a + b \).

The Hurwicz criterion estimates the resulting value as \( u_{ab} = a + b \).

In the general case, the values \( u_a \) and \( u_b \) are indeed different.

Explaining "Telescoping Effect" - That Time Perception Is Biased

People usually underestimate time passed since distant events, and overestimate for recent events.

Time is related to utility via discounting: \( m = u' q \).

This utility value is always in \([0, q]\).

We only know utility with some accuracy \( u \).

Instead of the original value \( u = u' q \), we only know that \( u' Q = q \).

For small \( u \), \( u' \approx q \).

Thus, we have the interval \([u, q]\), and Hurwicz method leads to the value:

\[ u + u' = u + q \text{ while } u + u' = u + (u' q - u') \text{ if } u' \approx q \]

Thus, for small \( u \), we have \( u' = q \).

The perceived time \( t \) comes from \( u = u' q \), so \( t \leq u \).

For large \( u \), we have \( u' = q \), so \( u = u' q \).

Hurwicz method leads to the value:

\[ u + u' = u + q \text{ while } u + u' = u + (u' q - u') \text{ if } u' \approx q \]

Thus, for large \( u \), we have \( u' = q \).

This explains the telescoping effect.

Future Plans: Theory

In terms of theoretical analysis, what we have done so far is based on deterministic decision making.

In practice, our decisions are often probabilistic.

In the same situation, we may select different alternatives, with different probabilities.

This situation has been analyzed in decision theory by a Nobelist D. McFadden.

However, his analysis assumes that we know the exact gains related to different alternatives.

In practice, we usually know the expected gains only with some uncertainty.

So, our main theoretical research would be to extend McFadden’s case to the use of uncertainty.

Future Plans: Explanations

First, there are still seemingly counterintuitive aspects of human behavior that need explaining, e.g.:

- an often cited phrase that giving is better than receiving;
- seems to be inconsistent with the usual utilitarian models of this behavior.

Second, the Hurwicz analysis does not explain why some people are more optimistic.

It is therefore desirable to try to understand this.

For this purpose, we will analyze which type of behavior works best in different situations.

Finally, it is desirable to look:

- not just at the results of human decision making, but also at procedures that humans use to reach their results.

For example, as part of these procedures, humans perform some non-traditional approximate computations.

We plan to analyze how these unusual procedures can be explained by decision making under uncertainty.

Acknowledgements

This work was supported in part by the US National Science Foundation via grant BHR-1247122 (Cyber-Share).