Astronomical Tests of Relativity: beyond Parameterized Post-Newtonian Formalism (PPN), to Testing Fundamental Principles

Vladik Kreinovich
University of Texas at El Paso
vladik@utep.edu
http://www.cs.utep.edu/vladik

1. Relativistic Celestial Mechanics: Current Status and Related Interesting Opportunity

- *Starting 1919:* experimentally compare general relativity (GRT) with Newton's mechanics.
- *1960s:* compare different relativistic gravitational theories, e.g., the Brans-Dicke Theory.
- *1970s:* Parameterized Post-Newtonian Formalism (PPN).
- *Current status:* all the observations have confirmed General Relativity (GRT).
- *Challenges.* GRT needs to be reconciled with:
  - quantum physics (into quantum gravity);
  - numerous surprising cosmological observations.
- *Idea:* prepare extended PPN, to test possible quantum- and cosmology-related modifications of GRT.

2. Towards Extended Post-Newtonian Formalism (EPN)

- *Idea:* prepare extended PPN, to test possible quantum- and cosmology-related modifications of GRT.
- *Details:* include the possibility of violating fundamental principles
  - that underlie the PPN formalism but
  - that may be violated in quantum physics.
- These *fundamental principles* include:
  - T-invariance,
  - P-invariance,
  - scale-invariance,
  - energy conservation,
  - spatial isotropy, etc.
- *Plan:* we present the first attempt to design the corresponding extended PPN formalism.

3. Possible Violations of T-Invariance

- Possible non-T-invariant terms PN terms in metric:
$$\delta g_{00} = \delta_1 \cdot \sum \frac{m_a \cdot (\vec{e}_a \cdot \vec{v}_a)}{r_a}, \quad \delta g_{0j} = \delta_2 \cdot \sum \frac{m_a \cdot e_{a,j}}{r_a}.$$
- *Fact:* light is determined by c^{-2} terms in $g_{\alpha\beta}$.
- *Corollary:* no effect on light.
- *Additional coord. transf.:* $x'_0 = x_0 + \alpha \cdot \sum m_a \cdot \ln(r_a)$.
- *Change in metric:* $\delta'_1 = \delta_1 + 2\alpha$, $\delta'_2 = \delta_2 + \alpha$.
- *Corollary:* T-invariant $\Leftrightarrow \delta_1 = 2\delta_2$.
- Lagrange function exists $\Leftrightarrow$ T-invariant.
- Motion Lorentz-invariant $\Leftrightarrow$ T-invariant.
- *Conclusion:* ether-dependent.
- *Perihelion shift* per rotation doesn't depend on m_a, r_a.
- *Restricted 3-body problem:* no effects modulo m^2.

4. T-Non-Invariance w/o Scale Invariance

- *General formula:* $\vec{a} = \vec{f}(m_a, \vec{r}, \vec{r}_a, \vec{v}, \vec{v}_a)$.
- *Requirements:* rotation-invariant; $\vec{f} = 0$ when $m_a = 0$.
- *Additional requirement:* energy conservation (impossible to have a closed cycle and gain some work).
- *1st conclusion:* radial motion in a central field is T-invariant.
- *Second conclusion:* under P-invariance, circular motion in a central field is T-invariant.
- *Fact:* for planets, orbits are almost circular.
- *Conclusion:* P-invariance $\Rightarrow$ T-invariance (mod. e).
- *Additional assumption:* $\vec{f}$ analytical w.r.t. m_a, $\vec{v}$, and $\vec{v}_a$, and Lorentz-covariant.
- *Conclusion:* the effect of non-T-invariant terms is c^{-5}, negligible in post-Newton approximation.

5. Possible Violations of P-Invariance

- *Most general term:* $\delta g_{0j} = \varepsilon \cdot \sum \frac{m_a}{r_a^2} \cdot (\vec{v}_a \times \vec{r}_a)_j$.
- *Observation:* all P-asymmetric terms are T-invariant.
- *Conclusion:* PT-invariance implies P- and T-invariance.
- *Fact:* no new coordinate transformations.
- Lagrange function exists $\Leftrightarrow$ P-invariant.
- Motion Lorentz-invariant $\Leftrightarrow$ P-invariant.
- *Perihelion effects* with $|\vec{w}| \approx 700$ km/s lead to

$$|\delta_1 - 2\delta_2| < 3 \cdot 10^{-7} \text{ and } |\varepsilon| \leq 0.01.$$

- *Comment:* discrete asymmetry is compatible with general covariance.
- *Example:* $L = L_1 + L_2$, where L_1 is a scalar and L_2 is a pseudo-scalar.

6. Possible Violations of P-Invariance (cont-d)

- Secular effects in the 2-body problem:

$$\frac{da}{dt} = \frac{de}{dt} = \frac{d\mathcal{M}}{dt} = 0;$$

$$\frac{di}{dt} = \varepsilon \cdot \frac{m}{a^2\sqrt{1-e^2}} \cdot (w_x \cdot \cos(\Omega) + w_y \cdot \sin\Omega);$$

$$\frac{d\Omega}{dt} = -\varepsilon \cdot \frac{m}{a^2\sqrt{1-e^2}} \cdot (\cot(i)(w_x \cdot \sin(\Omega) - w_y \cdot \cos(\Omega)) - w_z);$$

$$\frac{d\omega}{dt} = \varepsilon \cdot \frac{m}{a^2\sqrt{1-e^2}} \cdot (\cot(i) \cdot \cos(i) \cdot (w_x \sin\Omega - w_y \cos\Omega) - w_z \cdot \cos(i)).$$

- The effects are of the usual form $\frac{m}{a^2}$.
- *Conclusion:* $\varepsilon \leq$ accuracy of measuring perihelion shift, i.e., $|\varepsilon| \leq 0.01$.

7. Possible Violations of Equivalence Principle and Their Relation to Non-Conservation of Energy

- *General idea:* $\vec{F}_1 = m_1^I \cdot \vec{a}_1 = -G \cdot \dfrac{m_1^P \cdot m_2^A}{r_{12}^3} \cdot \vec{r}_{12}$.
- *Question:* what if energy is preserved?
- *Experiment:* connect 2 bodies by a rod; the system moves with force $\vec{F} = \vec{F}_1 + \vec{F}_2 \sim (m_1^P \cdot m_2^A - m_2^P \cdot m_1^A)$.
- If $\vec{F} \neq 0$, we can get energy out of nothing.
- $\vec{F} = 0 \Rightarrow m^A \propto m^P \Rightarrow m_1^I \cdot \vec{a}_1 = -G \cdot \dfrac{m_1^A \cdot m_2^A}{r_{12}^3} \cdot \vec{r}_{12}$.
- *Annihilation:* $a + \widetilde{a} \leftrightarrow 2\gamma$.
- *C-symmetry:* $m_a = m_{\widetilde{a}}$.
- *Experiments:* we let $a + \widetilde{a}$ move, then annihilate them, and let photons move back.
- *Conclusion:* if $m^I \not\propto m^A$, energy is not preserved.

8. Possible Cosmological Effects

- *Traditional PPN:* flat background metric $g_{\alpha\beta} = \eta_{\alpha\beta}$.
- *Cosmological terms:* $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{ij} + a_{\alpha\beta\gamma}x^{\gamma} + \dots$
- *Order of magnitude:* $a_{\alpha\beta\gamma}x^{\gamma} \approx r/R$, where r is Solar system, R is of cosmological order.
- *Conclusion:* safely ignore quadratic terms.
- *Combining with PPN:*

$$g_{\alpha\beta} = g_{\alpha\beta}^{\mathrm{PPN}} + h_{\alpha\beta} + a_{\alpha\beta\gamma}x^{\gamma}.$$

- *Effect on restricted 2-body problem:*

$$L = \frac{ds}{dt} = \sqrt{g_{\alpha\beta}\frac{dx^{\alpha}}{dt}\frac{dx^{\beta}}{dt}}.$$

- *Analysis:* main term is $\Delta L = 2a_{0ij}x^{i}v_{j}$.
- *Conclusion:* modulo full time deriv. $\Delta L \sim \vec{b}\cdot(\vec{v}\times\vec{x})$.
- *Resulting force:* magnetic-like $\vec{F} = 2\vec{b}\times\vec{v}$.

9. Possible Effects of Torsion

- *General idea:*

$$T^{\alpha\beta}_{|\gamma} = T^{\alpha\beta}_{;\gamma} + T^{\alpha\delta}S^{\delta}_{\delta\beta} + T^{\delta\beta}S^{\alpha}_{\delta\beta} = 0.$$

- *Due to asymmetry:* $T^{\alpha\beta}_{;\gamma} + T^{\alpha\delta}S_{\beta} = 0$, where $S_{\beta} \stackrel{\text{def}}{=} S^{\delta}_{\delta\beta}$.
- *General PPN-type dependence:*

$$S_0 = \beta_T \cdot \sum \frac{m_a \cdot (\vec{e}_a \cdot \vec{v}_a)}{r_a^2}; \quad S_i = \beta_T \cdot \sum \frac{m_a \cdot e_{ai}}{r_a^2}.$$

- *Additional* T-non-invariant and P-non-invariant terms are also possible.
- *Interesting conclusion:* we have a class of theories including Newton's gravity and intermediate theories.
- *Corollary:* we can simplify computations, since one term is Netwonian.

10. Finsler (Non-Riemannian) Space-Time

- *General formula:*

$$ds^2 = g_{00}\left(dx^0\right)^2 + 2g_{0i}dx^0dx^i + g_{ij}dx^idx^j + g_{ijk}\frac{dx^idx^jdx^k}{dx^0} + \ldots$$

- *Main effect:* on light

$$v^2 = 1 + \alpha_0 \cdot \frac{m}{r} + \alpha_1 \cdot \frac{m}{r} \cdot (\vec{e}\cdot\vec{k}) + \alpha_2 \cdot \frac{m}{r} \cdot (\vec{e}\cdot\vec{k})^2 + \ldots$$

- *Possible PPN-style generalization:*

$$\vec{a} = -\frac{m}{r^2} \cdot \vec{e} \cdot (1 + \gamma + a_1 \cdot (\vec{e}\cdot\vec{k}) + a_2 \cdot (\vec{e}\cdot\vec{k})^2 + \ldots) -$$
$$\frac{m}{r^2} \cdot \vec{k} \cdot (b_0 + b_1 \cdot (\vec{e}\cdot\vec{k}) + b_2 \cdot (\vec{e}\cdot\vec{k})^2 + \ldots) -$$
$$\frac{m}{r^2} \cdot (\vec{e}\times\vec{k}) \cdot (c_0 + c_1 \cdot (\vec{e}\cdot\vec{k}) + c_2 \cdot (\vec{e}\cdot\vec{k})^2 + \ldots)$$

- T-asymmetric terms: a_{2k+1}, b_{2n}, c_{2n}.
- P-asymmetric terms: c_0, c_1, c_2, $\ldots$

11. Acknowledgments

This work was supported in part by:

- by National Science Foundation grant HRD-0734825, and
- by Grant 1 T36 GM078000-01 from the National Institutes of Health.