Identification of Human Gait in Neuro-Rehabilitation: Towards Efficient Algorithms

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1. Introduction

- *Fact:* many neurological diseases drastically decrease the patient's ability to walk w/o physical assistance.
- *Examples:* stroke, traumatic body injury, and spinal cord injury
- *Extensive rehabilitation* is needed to re-establish normal gait.
- *At present:* rehabilitation requires gait assessment by highly qualified experienced clinicians.
- *Problem:* difficult to access, high costs.
- *It is desirable:* to automate gait assessment:
  - to make rehabilitations easier to access, and
  - to decrease the rehabilitation cost.

2. Formulation of the Problem in Precise Terms

- A gait is measured by the dependence $x'(t)$ of some characteristic on time.
- *Example:* the acceleration or the angle between different parts of the foot.
- *The gait assessment* means comparing
  - the recorded patient's gait with
  - a standard (average) gait $x(t)$ of healthy people of the same age, body measurements, etc.
- *Problem:* patients walk slower.
- *Solution:* appropriately shift and "scale" the standard gait.
- *Resulting formulation:* find the values t_0 and λ for which

$$x'(t) \approx x(\lambda \cdot t - t_0).$$

3. Straightforward Algorithm and Its Limitations

- *Given:* the patient gait $x'(t)$ and the standard gait $x(t)$.
- *Find:* the values t_0 and λ for which

$$x'(t) \approx x(\lambda \cdot t - t_0).$$

- *Straightforward idea:* try all possible shifts and scalings.
- *Limitations:* this is computationally very intensive.
- *Objective:* to design an efficient algorithm for finding the optimal combination of a shift and a scaling.
- *Our idea:* adjust the known image referencing techniques that use Fast Fourier Transform.

4. Why Fourier-Based Methods

- *Simplest case:* find the shift t_0 for which $x'(t) \approx x(t - t_0)$.
- *Notation:* let n be the number of moments of time for which we know $x(t)$.
- *Natural formalization:* least squares method – find t_0 that minimizes $I \stackrel{\text{def}}{=} \int (x'(t) - x(t - t_0))^2\,\mathrm{d}t$.
- *Simplification:*
$$I = \int (x'(t))^2\,\mathrm{d}t + \int x(t-t_0)^2\,\mathrm{d}t - 2\int x'(t)\cdot x(t-t_0)\,\mathrm{d}t.$$
- *Analysis:* the first two terms do not depend on t_0.
- *Conclusion:* find t_0 for which the convolution $J(t_0) \stackrel{\text{def}}{=} \int x'(t)\cdot x(t-t_0)\,\mathrm{d}t$ is the largest.
- *Computation time:* we need n convolutions, with n steps each; overall time $O(n)\cdot O(n) = O(n^2)$.

5. Why Fourier-Based Methods (cont-d)

- *Fact:* convolution is one of the main techniques in signal processing.
- *Fact:* we can compute convolution $J(t_0)$ faster:
  - first, we apply FFT to the original signals, resulting in functions $F(\omega)$ and $F'(\omega)$;
  - then, for each frequency ω, we compute the product

    $$P(\omega) \stackrel{\text{def}}{=} F(\omega) \cdot (F')^*(\omega);$$

  - third, we apply FFT^{-1} to the resulting function $P(\omega)$, and get the desired convolution $J(t_0)$.
- Finally, we find t_0 for which $J(t_0) \to \max$.
- FFT requires $O(n \cdot \log(n))$ steps, multiplication and search for t_0 is $O(n)$.
- So, we find t_0 in time $O(n \cdot \log(n)) + O(n) = O(n \cdot \log(n)) \ll O(n^2)$.

6. Shift Detection: Resulting Algorithm

- *Ideal case:* $x'(t) = x(t - t_0)$, hence:
  - $F'(\omega) = e^{2\pi\cdot\mathrm{i}\cdot(-\omega\cdot t_0)} \cdot F(\omega)$;
  - here, the ratio $R(\omega) \stackrel{\text{def}}{=} P(\omega)/|P(\omega)|$ is equal to $R(\omega) = e^{2\pi\cdot\mathrm{i}\cdot(-\omega\cdot t_0)}$;
  - thus, the FFT^{-1} of $R(\omega)$ is equal to $I(t) = \delta(t+t_0)$;
  - so, t_0 is the only value for which $I(-t) \neq 0$.
- *In practice:* $x'(t) \approx x(t - t_0)$, so:
  - we apply FFT to the original signals $x(t)$, $x'(t)$ and compute their Fourier transforms $F(\omega)$ and $F'(\omega)$;
  - we compute the product $P(\omega) = F(\omega) \cdot (F')^*(\omega)$ and the ratio $R(\omega) = P(\omega)/|P(\omega)|$;
  - we apply FFT^{-1} to $R(\omega)$ and get $I(t)$;
  - we find t_0 for which $|I(-t_0)| \to \max$.

7. General Case

- *General case:* $x'(t) \approx x(\lambda \cdot t - t_0)$.
- *Analysis:* the magnitudes $M(\omega) = |F(\omega)|$ and $M'(\omega) = |F'(\omega)|$ differ by scaling: $M'(\omega) \approx (1/\lambda) \cdot M(\omega/\lambda)$.
- *Idea:* in *log frequencies* $\rho = \log(\omega)$, scaling becomes shift-like: $\rho \to \rho - b$, where $b = \log(\lambda)$.
- *Resulting algorithm:*
  - transform $M(\omega)$ and $M'(\omega)$ to log frequencies;
  - use the above FFT-based algorithm to determine the corresponding shift $\log(\lambda)$;
  - from the corresponding "shift" value, reconstruct the scaling coefficient λ;
  - re-scale $x(t)$ to $x(\lambda \cdot t)$ and use the same FFT-based algorithm to compute the shift t_0.

8. Conclusions

- Many neurological diseases drastically decrease the patient's ability to walk without physical assistance.
- To re-establish normal gait, patients undergo extensive rehabilitation.
- At present, rehabilitation requires gait assessment by highly qualified experienced clinicians.
- To make rehabilitations easier to access, it is desirable to automate gait assessment.
- In this paper, we design a fast algorithm that uses Fast Fourier Transform for gait assessment.

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