Plans Are Worthless but Planning Is Everything: A Theoretical Explanation of Eisenhower’s Observation

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1. Eisenhower’s Observation

- Dwight D. Eisenhower was:
  - the Supreme Commander of the Allied Expeditionary Forces in Europe during WW2
  - and later the US President.
- He emphasized that his war experience taught him that “plans are worthless, but planning is everything”.
- At first glance, this sounds paradoxical: if plans are worthless, why bother with planning at all?
- In this paper, we show that this Eisenhower’s observation has a meaning:
  - while following the original plan in constantly changing circumstances is often not a good idea,
  - the existence of a pre-computed original plan enables us to produce an almost-optimal strategy.
2. **Rational Decision Making: a Brief Reminder**

- According to decision making theory:
  - decisions by a rational decision maker
  - can be described as maximize the value a certain function known as utility.

- E.g., in financial situations, when a company needs to make a decision, the overall profit can be used as utility.

- To describe a possible action $x$, we usually need to describe the values of several quantities $x_1, \ldots, x_n$.

- E.g., a decision about a plant involves selecting amounts $x_i$ of manufactured gadgets of different type.

- Similarly, we need several quantities $a_1, \ldots, a_m$ to describe a situation.

- Let $u(x, a)$ denote the utility that results from performing action $x$ in situation $a$. 
3. In These Terms, What Is Planning

- Let \( \tilde{a} \) describe the original situation.
- Based on this situation, we come up with an action \( \tilde{x} \) that maximizes the corresponding utility:

\[
u(\tilde{x}, \tilde{a}) = \max_x u(x, \tilde{a}).\]

- Computing this optimal action \( \tilde{x} \) is what we usually call planning.
- When we need to start acting, the situation may have changed to \( a \neq \tilde{a} \).
- Let us denote the corresponding change by \( \Delta a \overset{\text{def}}{=} a - \tilde{a} \), then \( a = \tilde{a} + \Delta a \).
4. Options

- One possibility is to simply ignore the change, and apply the original plan $\tilde{x}$ to the new situation $a = \tilde{a} + \Delta a$.
- This plan is, in general, not optimal for the new situation.
- The actually optimal plan is $x^{opt}$ for which
  \[ u(x^{opt}, \tilde{a} + \Delta a) = \max_x u(x, \tilde{a} + \Delta a). \]
- In comparison with the optimal plan, we lose the amount $L_0 \overset{\text{def}}{=} u(x^{opt}, \tilde{a} + \Delta a) - u(\tilde{x}, \tilde{a} + \Delta a)$.
- Why cannot we just find the optimal solution for the new situation?
- Optimization is NP-hard, so, it is not possible to find the exact optimum in reasonable time.
5. Options (cont-d)

- What we can do is:
  - try to use some feasible algorithm – e.g., solving a system of linear equations,
  - to modify the plan $\tilde{x}$ into $\tilde{x} + \Delta x$.

- Due to NP-hardness, this feasibly modified plan is, in general, not optimal.

- We hope that the resulting loss $L_1 \overset{\text{def}}{=} u(x^{\text{opt}}, \tilde{a} + \Delta a) - u(\tilde{x} + \Delta x, \tilde{a} + \Delta a)$ is much smaller than $L_0$.

- In this paper, we show that indeed $L_1 \ll L_0$; so:
  - even if $L_0$ is so large that the original plan is worthless,
  - the modified plan may leads to a reasonably small loss $L_1 \ll L_0$.

- This explains Eisenhower’s observation.
6. Estimating $L_0$

- We assume that the difference $\Delta a$ is reasonably small.
- So, the corresponding difference in action $\Delta x^{\text{opt}} \overset{\text{def}}{=} x^{\text{opt}} - \tilde{x}$ is also small.
- We can therefore expand $L_0$ in Taylor series and keep only terms linear and quadratic in $\Delta x$:

$$L_0 = u(x^{\text{opt}}, \tilde{a} + \Delta a) - u(x^{\text{opt}} - \Delta x^{\text{opt}}, \tilde{a} + \Delta a) =$$

$$\sum_{i=1}^{n} \frac{\partial u}{\partial x_i}(x^{\text{opt}}, \tilde{a} + \Delta a) \cdot \Delta x_i^{\text{opt}} +$$

$$\frac{1}{2} \cdot \sum_{i=1}^{n} \sum_{i' = 1}^{n} \frac{\partial^2 u}{\partial x_i \partial x_{i'}}(x^{\text{opt}}, \tilde{a} + \Delta a) \cdot \Delta x_i^{\text{opt}} \cdot \Delta x_{i'}^{\text{opt}} + o((\Delta a)^2).$$

- By definition, the action $x^{\text{opt}}$ maximizes $u(x, \tilde{a} + \Delta a)$.
- Thus, we have $\frac{\partial u}{\partial x_i}(x^{\text{opt}}, \tilde{a} + \Delta a) = 0$. 
7. Estimating $L_0$ (cont-d)

- So, the above expression for $L_0$ takes the simplified form

$$L_0 = \frac{1}{2} \sum_{i=1}^{n} \sum_{i'=1}^{n} \frac{\partial^2 u}{\partial x_i \partial x_{i'}} (x^{\text{opt}}, \tilde{a} + \Delta a) \cdot \Delta x^{\text{opt}}_i \cdot \Delta x^{\text{opt}}_{i'} + o((\Delta a)^2).$$

- $\Delta x^{\text{opt}}_i$ can be estimated from the condition:

$$\frac{\partial u}{\partial x_i}(x^{\text{opt}}, \tilde{a} + \Delta a) = \frac{\partial u}{\partial x_i}(\tilde{x} + \Delta x^{\text{opt}}, \tilde{a} + \Delta) = 0.$$

- For $a = \tilde{a}$, $u$ is max when $x = \tilde{x}$, so $\frac{\partial u}{\partial x_i}(\tilde{x}, \tilde{a}) = 0$.

- Expanding the equation in Taylor series in $\Delta x_i$ and $\Delta a_j$ and taking $\frac{\partial u}{\partial x_i}(\tilde{x}, \tilde{a}) = 0$ into account, we get:

$$\sum_{i'=1}^{n} \frac{\partial^2 u}{\partial x_i \partial x_{i'}} (\tilde{x}, \tilde{a}) \cdot \Delta x^{\text{opt}}_{i'} + \sum_{j=1}^{m} \frac{\partial^2 u}{\partial x_i \partial a_j} (\tilde{x}, \tilde{a}) \cdot \Delta a_j + o(\Delta x, \Delta a) = 0.$$
8. Estimating $L_0$ (final)

- Thus, the first approximation $\Delta x_i$ to the values $\Delta x_{i}^{\text{opt}}$ satisfies a system of linear equations:

$$\sum_{i'=1}^{n} \frac{\partial^2 u}{\partial x_i \partial x_{i'}}(\tilde{x}, \tilde{a}) \cdot \Delta x_j = -\sum_{j=1}^{m} \frac{\partial^2 u}{\partial x_i \partial a_j}(\tilde{x}, \tilde{a}) \cdot \Delta a_j.$$

- A solution to a system of linear equations is a linear combination of the right-hand sides.

- Thus, the values $\Delta x_i$ are a linear function of $\Delta a_j$.

- Substituting these linear expressions into the formula for $L_0$, we conclude that $L_0$ is quadratic in $\Delta a_j$: 

$$L_0 = \sum_{j=1}^{m} \sum_{j'=1}^{m} k_{jj'} \cdot \Delta a_j \cdot \Delta a_{j'} + o((\Delta a)^2) \text{ for some } k_{jj'}.$$
9. **Estimating L₁**

- The 1st approximation $\Delta x$ to the difference $\Delta x^{\text{opt}}$ can be obtained by solving a system of linear equations.
- How much do we lose if we use $x^{\text{lin}} = \tilde{x} + \Delta x$?
- Here, $\Delta x^{\text{opt}} = \Delta x + \delta x$, where $\delta x$ is of 2nd order in $\Delta x$ and $\Delta a$: $\delta x = O((\Delta a)^2)$.
- The loss $L₁$ of using $x^{\text{lin}} = x^{\text{opt}} - \delta x$ instead of $x^{\text{opt}}$ is:
  \[
  L₁ = u(x^{\text{opt}}, \tilde{a} + \Delta a) - u(x^{\text{lin}}, \tilde{a} + \Delta a) = \\
  u(x^{\text{opt}}, \tilde{a} + \Delta a) - u(x^{\text{opt}} - \delta x, \tilde{a} + \Delta a).
  \]
- If we expand this expression in $\delta x$, we get:
  \[
  L₁ = \sum_{i=1}^{n} \frac{\partial u}{\partial x_i}(x^{\text{opt}}, \tilde{a} + \Delta a) \cdot \delta x_i + \\
  \frac{1}{2} \cdot \sum_{i=1}^{n} \sum_{i'=1}^{n} \frac{\partial^2 u}{\partial x_i \partial x_{i'}}(x^{\text{opt}}, \tilde{a} + \Delta a) \cdot \delta x_i \cdot \delta x_{i'} + o((\delta x)^2).
  \]
10. Estimating $L_1$ (cont-d)

- Since $x^{\text{opt}}$ is the action that, for $a = \tilde{a} + \Delta a$, maximizes utility, we get $\frac{\partial u}{\partial x_i}(x^{\text{opt}}, \tilde{a} + \Delta a) = 0$.

- Thus, the expression for $L_1$ gets a simplified form

$$L_1 = \frac{1}{2} \cdot \sum_{i=1}^{n} \sum_{i' = 1}^{n} \frac{\partial^2 u}{\partial x_i \partial x_{i'}}(x^{\text{opt}}, \tilde{a} + \Delta a) \cdot \delta x_i \cdot \delta x_{i'} + o((\delta x)^2).$$

- We know that the values $\delta x_i$ are quadratic in $\Delta a$.

- Thus, we conclude that for the modified action, the loss $L_1$ is a 4-th order function of $\Delta a_j$:

$$L_1 = \sum_{j=1}^{m} \sum_{j'=1}^{m} \sum_{j''=1}^{m} \sum_{j'''=1}^{m} k_{jj'j''j'''} \cdot \Delta a_j \cdot \Delta a_{j'} \cdot \Delta a_{j''} \cdot \Delta a_{j'''} + o((\Delta a)^5).$$
11. Conclusions

- We conclude that:
  - the loss $L_0$ related to using the original plan is quadratic in $\Delta a$, while
  - the loss $L_1$ related to using a feasibly modified plan is of 4th order in terms of $\Delta a$.

- For small $\Delta a$, we have $L_1 \sim (\Delta a)^4 \ll L_0 \sim (\Delta a)^2$.

- Let $\varepsilon > 0$ be the maximum loss that we tolerate.

- Since $L_1 \ll L_0$, we have three possible cases:
  
  (1) $\varepsilon < L_1$, (2) $L_1 \leq \varepsilon \leq L_0$, and (3) $L_0 < \varepsilon$.

- In the 1st case, even the modified action does not help.

- In the 3rd case, the change in the situation is so small that it is Ok to use the original plan $\tilde{x}$.
12. Conclusions (cont-d)

- In the second case, we have exactly the Eisenhower situation:
  - if we use the original plan $\tilde{x}$, the resulting loss $L_0$ much larger than we can tolerate;
  - in this sense, the original plan is worthless;
  - on the other hand, if we feasible modify the original plan into $x^{\text{lin}}$, then we get an acceptable action.
- So, we indeed get a theoretical justification of Eisenhower’s observation.
13. Acknowledgments

- This work was supported in part by the National Science Foundation grants:
  - HRD-0734825 and HRD-1242122 (Cyber-ShARE Center of Excellence) and
  - DUE-0926721, and
- by an award from Prudential Foundation.