Towards Optimal Sensor Placement in Multi-Zone Measurements

Octavio Lerma, Craig Tweedie, and Vladik Kreinovich

Cyber-ShARE Center
University of Texas at El Paso
El Paso, TX 79968
1. Outline

- In multi-zone areas, boundaries change with time.
- It is desirable to place sensors in such a way that the boundary is covered at all times.
- In this talk, we describe the optimal sensor placement with this property.
- In this optimal placement, sensors are placed along a see-saw trajectory between
  - the current location of the boundary and
  - its farthest future location.
2. Need for Measurements

- In many areas, we know the partial differential equations that can be used for the prediction.
- Example: weather prediction.
- To make accurate predictions, we need to have a very accurate picture of the initial conditions.
- This picture must describe current values of
  - temperature,
  - atmospheric pressure,
  - wind,
  - and other characteristics
  at different spatial locations.
- To measure the values of these characteristics, we need to place sensors at different locations.
3. Need for Sensor Placement

- In some areas – e.g., in and around big cities – there is usually a large number of sensors.
- Many of the sensors operated by volunteers who place these sensors in their homes.
- However, in other areas (e.g., in the Arctic), the existing sensor coverage is too sparse.
- So, more sensors are needed.
- The placement of new sensors
  - not only helps in achieving short-term goals such as weather predictions,
  - it also helps in analyzing long-term effects such as climate and environmental changes.
4. Need for Optimal Sensor Placements

- Placement and maintenance of sensors in remote areas is often costly.
- So, it is desirable to come up with the optimal ways to place sensors – so that we can achieve
  - the desired accuracy and
  - coverage at the smallest possible cost.
- The problem of optimal sensor placement is simpler for homogeneous (single-zone) regions.
- In these regions, the measured quantity smoothly changes from one location to another.
- The $i$-th sensor measures the value $v_i = v(x_i)$ at its location $x_i$.
- For locations $x$ close to $x_i$, we have $v(x) \approx v(x_i) = v_i$. 
5. Optimal Sensor Location: Case of Single-Zone (Homogeneous) Regions

- Suppose that we have placed sensors at locations $x_1, \ldots, x_n$.
- For every spatial location $x$, we approximate $v(x)$ by the result $v(x_i)$ measured by the closest sensor $i$.
- The quality of a sensor network is measured by accuracy with which it determines $v(x)$ at all $x$.
- From this viewpoint, the quality of a sensor network is determined by the “worst” spatial location $x_0$.
- $x_0$ is a spatial location which is the farthest away from all the sensors.
- For location $x_0$, the approximation accuracy is the worst.
- Thus, for homogeneous regions, the optimal sensor placement is uniform.
6. Case of Multi-Zone Measurements

- In practice, regions are often not homogeneous.
- Regions consist of several distinct zones with a sharp boundary between the zones.
- The simplest example is a shoreline – the boundary between the land and the ocean.
- In mountain regions, there is also a sharp line between a glacier and the grassy zone around it, etc.
- For such multi-zone areas,
  - we need not only to find the values of the desired characteristics at different zones,
  - we also need to get a good understanding of the exact location of the boundary between the zones.
- In practice, the boundaries change.
- It is very important to trace these changes.
7. How to Optimally Place Sensors Near the Inter-Zone Boundary

- We assume that the boundary is reasonable smooth.
- So, for some reasonably large (and known) value $\ell$,
  - if we place sensors at distance $\ell$ from each other,
  - then we will get a pretty good picture of the whole boundary.

- Each sensor is located at a distance $\ell$ from the previous one.
- We need to cover the boundary of total length $L$.
- So, we need $L/\ell$ sensors.
8. Case of Moving Boundary

- We must cover not only the current boundary, but also its future locations.
- So, we need to place several lines of sensors.
- Each sensor line requires $L/\ell$ sensors.
- Let $N$ denote the total number of sensors that our budget can afford.
- Thus, we can place $k = N/(L/\ell)$ sensor lines.
- Let $V_0$ be the speed with which the boundary moves.
- Let $T$ be the planned lifetime of the sensor network.
- Thus, we need to place $k$ sensor lines at distances from 0 to $D = V_0 \cdot T$.
- It is reasonable to select these distances to be equally spaced, at distance 0, $d = D/k$, $2 \cdot d$, $3 \cdot d$, …
9. How to Arrange Different Sensor Lines Relative to Each Other?

- On each sensor line, the sensors are equally spaced, with a distance $\ell$ between two neighboring sensors.
- Once we determine a place for one of the sensors on this line, the location of others is determined.
- Namely, we place sensors on this line at distances $\ell$, $2 \cdot \ell$, $\ldots$, from this original sensor.
- Let us start with such an equally spaced arrangement of sensors on the original boundary.
- Let us pick two neighboring sensors at distance $\ell$ from each other.
- For each sensor line, we can then select the segment parallel to the segment between these two sensors.
10. Sensor Placement (cont-d)

- On a segment of length $\ell$, each sensor line has exactly one sensor.
- Once the locations of all these sensors is fixed, the location of all other sensors is uniquely determined.
- On each sensor line, we place sensors on this line at distances $\ell, 2\cdot \ell, \ldots$, from the sensor from this segment.
- Thus, to fully determine the sensor configuration, we must decide how to place sensors within this segment.
11. How to Place Sensors Within a Segment: Selecting an Optimal Path

- When we place sensors, we need to physically travel from one sensor to the next one.
- We are talking about sensors in a remote area, where travel is difficult.
- We must thus minimize the total length of the path connecting all these sensors.
- Similar minimization is needed for maintenance.
- Let us consider the path that
  - starts with a sensor $S$ at the original boundary and
  - ends us at the next sensor $S'$ on this boundary.
- We also need to visit sensors which are farthest away
  - at distance $D$ – from the original boundary.
12. How to Place Sensors (cont-d)

- Thus, our sensor-visiting path must
  - start from a point on the original boundary,
  - go to a point $F$ on the line at distance $D$,
  - and then go back:

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- - - - -   F   - - - - -
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- Out of all paths with this property, we must select the shortest one.
13. Analysis of the Problem

• Once the points $S$ and $F$ are fixed, then the shortest path from $S$ to $F$ is the straight line.

• Similarly, the shortest path from $F$ to $S'$ is also a straight line.

• Thus, the shortest path must consist of two straight-line segments: from $S$ to $F$ and from $F$ to $S'$:
14. How to Select the Optimal Location of the Farthest Sensor

We want to find the optimal location of the sensor $F$. Thus, we must minimize the overall length $p$ of the path $SFS'$:

$$p = \sqrt{D^2 + x^2} + \sqrt{D^2 + (\ell - x)^2}.$$

Differentiating this expression and equating the derivative to 0, we conclude that $x = \ell/2$. 
15. The Resulting Optimal Path

- We start from the location \( S \).
- We take a straight line to a point \( F \) which is located:
  - on the farthest sensor line
  - midway between \( S \) and the next sensor \( S' \).
- From that \( F \), we take a straight line path back to \( S' \).
- etc.
16. Where to Place Sensors Along the Path? Problem

- Within each segment, the path intersect each sensor line twice.
- So, we can place a sensor either on the ascending or on the descending parts of the path:
17. Where to Place Sensors Along the Path? Solution

- As above, we place sensors as uniformly as possible.
- Thus, we can, e.g.,
  - place sensors on the even-numbered sensor lines on the ascending path, and
  - place sensors from the odd-numbered sensor lines on the descending path:
18. Conclusion: Assumptions and Objectives

- To maintain desired accuracy, we need to place sensors at distance at most $\ell$ along the boundary.

- During the sensor lifetime, the boundary will move by the distance $D$.

- Based on the available number of sensors, we place sensors
  - along $k$ sensor lines,
  - at distance $0, d = D/k, 2d, \ldots$, from the original boundary.

Our objectives are:

- to minimize the path that we need to traverse to place and to maintain the sensor, and

- to maintain the most accurate (hence, homogeneous) spatial coverage at any given moment of time.
19. Conclusion: Resulting Optimal Placement

To satisfy the above objectives:

- we place the sensor at equal distance $\ell$ along the original sensor boundary, and
- place other sensors along the see-saw path that goes
  - from every sensor on the original boundary
  - to the farthest (distance $D$) sensor line
  - and then back –
  
at the exact same angle;
- sensors from the even-numbered sensors are then placed on the ascending part of the sensor-connecting path;
- sensors from the odd-numbered sensors are then placed on the descending part of the sensor-connecting path.
20. Conclusion: Resulting Optimal Placement (cont-d)

Here is the resulting optimal see-saw configuration of the sensors:

Here:

• $\ell$ is the distance between the sensors along the boundary;

• $D$ is the depth to be covered;

• $d$ is the distance between two sensor lines.

Here:

\[ \begin{align*}
\ell & \quad \text{is the distance between the sensors along the boundary;} \\
D & \quad \text{is the depth to be covered;} \\
d & \quad \text{is the distance between two sensor lines.}
\end{align*} \]
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