High Concentrations Naturally Lead to Fuzzy-Type Interactions and to Gravitational Wave Bursts

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1. Equations Describing the Physical World

- Traditionally, our knowledge is described in precise terms, from Newton’s laws to relativity theory.

- In many physical situations, it is not possible to exactly predict the future state of a system.

- In some situations:
  - we know the exact equations describing the interaction of particles,
  - but the number of particles is so huge that it is not possible to exactly solve this system of equations.

- For example, a room full of air contains about $10^{23}$ molecules.

- In this case, all we can do is make predictions about the frequency of different outcomes.
2. Physical Equations (cont-d)

- In other words, we can only make predictions about the probabilities of different events.

- This is the case of *statistical physics*.

- If we take *quantum effects* into account, then the same phenomenon occurs for all possible physical processes.

- Indeed, according to quantum physics:
  - it is not even theoretically possible
  - to predict the exact future values of all the physical quantities such as coordinates, momentum, etc.

- All we can do is predict the corresponding probabilities.
3. Physical Equations Describing the Probabilities Are Also Exact

- While the knowledge is probabilistic, equations describing how these probabilities change are exact.
- This is true for Boltzmann’s equations of statistical physics and for Schroedinger’s quantum equations.
- These equations are usually smooth (differentiable).
- Different particles are reasonable independent.
- So the overall probability can be obtained by multiplying probabilities corresponding to different particles.
- The product function is differentiable infinitely many times.
- So usually, the corresponding equations are also differentiable (smooth).
4. Need to Describe Expert Knowledge

• In many real-life situations:
  – in addition to (or, sometimes, instead of) the exact equations,
  – we also have imprecise (“fuzzy”) expert knowledge,
  – knowledge that experts describe by using imprecise natural-language words.

• For example, a medical doctor may say that a skin irritation is suspicious if it has irregular shape.

• A medicine is recommended if the patient has a high fever.

• However, what exactly is irregular or high is not well-defined, it is subjective.
5. Fuzzy Logic as a Natural Way to Describe Imprecise Expert Knowledge

- In both above examples, it is not the case that we have an exact threshold on temperature, so that:
  - below this threshold, we have one decision, and
  - above the threshold, we have another decision.

- This would make no sense:
  - why give a medicine to someone whose body temperature is 39.00°C
  - but not to someone whose temperature is 38.99°C?

- For temperatures close to some transition value:
  - experts are not 100% sure whether the temperature is high (or whether the shape is irregular),
  - they are only confident to some degree.
6. Fuzzy Logic (cont-d)

- In the computer, “absolutely true” is usually represented as 1, and “absolutely false” as 0.

- So it is reasonable to describe intermediate degrees of confidence by intermediate numbers, from [0, 1].

- This is the main idea behind fuzzy logic.

- This formalism was invented by Lotfi A. Zadeh to describe imprecise expert knowledge.

- In fuzzy logic, to describe each imprecise property \( P \) like “high”, we assign,
  
  - to each possible value \( q \) of the corresponding quantity,
  - a number \( \mu(q) \) from the interval \([0, 1]\) that describes to what extent the expert is confident in \( P(q) \),
  - e.g., to what extent the given temperature \( q \) is high.
7. Fuzzy Degrees

• How can we estimate the expert’s degrees of confidence?

• We can, e.g., ask each expert to mark his/her degree of confidence on a scale from 0 to 10:
  – 0 meaning no confidence at all, and
  – 10 meaning absolutely sure.

• To get a value between 0 and 1, we divide the resulting estimate by 10.

• Experts can estimate degrees of certainty in their statements.

• However, conclusions based on expert knowledge often take into account several expert statements.
8. Fuzzy Degrees (cont-d)

• Our degree of confidence in such a conclusion is thus equal to our degree of confidence that:
  – the first of used statements is true and
  – the second used statement is true, etc.

• In other words,
  – in addition to the expert’s degrees of confidence in their statements \( S_1, \ldots, S_n \),
  – we also need to estimate the degrees of confidence in “and”-combinations \( S_i \& S_j, S_i \& S_j \& S_k \), etc.

• In the ideal world, we can ask the experts to estimate the degree of confidence in each such combination.

• However, this is not realistically possible.

• Indeed, for \( n \) original statements, there are \( 2^n - 1 \) such combinations.
9. Fuzzy Degrees (cont-d)

- Indeed, combinations are in 1-1 correspondence with non-empty subsets of the set of \( n \) statements.
- Already for reasonable \( n = 30 \), we get an astronomical number \( 2^{30} \approx 10^9 \) combinations.
- There is no way that we can ask a billion questions to the experts.
- We cannot elicit the expert’s degree of confidence in “and”-combinations directly from the experts.
- So, we need to estimate these degrees based on the experts’ degrees of confidence in each statement \( S_i \).
10. Fuzzy Degrees (cont-d)

- In other words, we need to be able:
  - to combine the degrees of confidence $a$ and $b$ of statements $A$ and $B$
  - into an estimate for degree of confidence in the “and”-combination $A \& B$.

- The algorithm for such combination is called an “and”-operation or, for historical reasons, a $t$-norm.

- The result of applying this combination algorithm to numbers $a$ and $b$ will be denoted $f_\&(a, b)$. 
11. How to Combine Fuzzy Degrees?

• Which operation \( f_\& (a, b) \) should we choose?

• First, since \( A \& B \) means the same as \( B \& A \), it is reasonable to require that the resulting estimates coincide:

\[
f_\& (a, b) = f_\& (b, a).\]

• Second, since \( A \& A \) means the same as \( A \), it is reasonable to require that \( f_\& (a, a) = a \).

• Since \( A \& B \) is a stronger statement than each of \( A \) and \( B \) (it implies both \( A \) and \( B \)),

\[
f_\& (a, b) \leq a \text{ and } f_\& (a, b) \leq b.\]
12. How to Combine Fuzzy Degrees (cont-d)

• Finally:
  – if our degree of confidence in one or both of the statements $A$ and $B$ increases,
  – the resulting degree of confidence in $A \& B$ should also increase – or at least remain the same:
    
    if $a \leq a'$ and $b \leq b'$, then $f_{\&}(a, b) \leq f_{\&}(a', b')$.

• It turns out that there is exactly one operation that satisfies these four properties: $f_{\&}(a, b) = \min(a, b)$.

• This operation, proposed in the very first of Zadeh’s papers, is indeed one of the most widely used ones.

• Indeed, it is easy to see that the min-operation satisfies all four above properties.

• Vice versa, let us assume that a function $f_{\&}(a, b)$ satisfies all four properties.
13. How to Combine Fuzzy Degrees (cont-d)

- Since this function is symmetric (first property), it is sufficient to consider the case when \( a \leq b \).
- In this case, due to the third property, \( f_\& (a, b) \leq a \).
- On the other hand, since \( a \leq b \), monotonicity implies that \( f_\& (a, a) \leq f_\& (a, b) \).
- By the 2nd property, \( f_\& (a, a) = a \), so \( a \leq f_\& (a, b) \).
- From \( f_\& (a, b) \leq a \) and \( a \leq f_\& (a, b) \), we conclude that \( f_\& (a, b) = a \) for \( a \leq b \).
- So, for \( a \leq b \), we have \( f_\& (a, b) = \min (a, b) \).
- Due to symmetry, this equality holds for \( a \geq b \) as well.
- The statement is proven.
14. Chemical Kinetics and Boltzmann’s Formulas of Statistical Physics: a Brief Reminder

- In many real-life situations, we have a large number of small interacting particles.
- These particles may be molecules of different type whose interaction constitute chemical reactions.
- These particles may be molecules of gas whose interaction simply means bouncing off each other, etc.
- The molecules interact when they are close to each other.
- The usual way to describe such an interaction takes into account that:
  - on the microscopic level, the space is mostly empty,
  - so such interactions are reasonably rare.
15. Chemical Kinetics (cont-d)

• As a result, e.g., in chemical reactions,
  – the probability that a molecule of the 1st type gets involved in the interaction in a given time period
  – is proportional to the concentration $b$ of the molecules of the 2nd type.

• To get the amount of interactions, we multiply this probability by the # of 2nd type molecules.

• This number is proportional to their concentration $a$.

• Thus, the intensity of intersection is proportional to the product $a \cdot b$: $\frac{da}{dt} = k \cdot a \cdot b + \ldots$

• When the reaction leads to physical motion, the change in the location $x$ is also $\sim a \cdot b$: $\frac{dx}{dt} = k \cdot a \cdot b + \ldots$
16. Case of High Concentration

• What happens when the concentration is high?

• Let us consider yet a phenomenon where the product model is applied – the predator-prey model.

• For example, if both wolves and rabbits are reasonably rare, it takes some running for a wolf to find a rabbit.

• The intensity of the wolves-eat-rabbits process is proportional to the product $a \cdot b$ of their concentrations.

• But what if the concentrations become large?

• In this case, each wolf is actively engaged in eating rabbits – as long as there is sufficient number of rabbits.
  
  – If the conc. of wolves $a$ is $\leq$ that of rabbits $b$,
  – the intensity of the eating process is proportional to the number of wolves, i.e., to $a$. 
17. Case of High Concentration (cont-d)

• On the other hand:
  – if there are fewer rabbits than wolves, i.e., if $a > b$,
  – then the intensity of eating is proportional to the number of rabbits, i.e., to $b$.

• In both cases, the intensity of interaction is proportional to min($a, b$):

$$\frac{da}{dt} = k \cdot \text{min}(a, b) + \ldots,$$
$$\frac{dx}{dt} = k \cdot \text{min}(a, b) + \ldots$$
18. Discussion

- The traditional product formula is similar to the formula for the probability of the combined event $A \& B$.
- The new formula resembles a formula for finding the fuzzy degree of confidence of such a combined event.
- Fuzzy-type interactions are not only useful for describing high-concentration physical phenomena.
- Since these interactions correspond to faster reactions, their simulation can speed up computations.
19. Can We Detect Fuzzy-Type Interactions?

- Sometimes, we are in the vicinity of high-concentration processes – e.g., in catalysis.
- Then, we can directly observe the fuzzy-type behavior.
- However, most physical processes with high concentrations are in the domain of astrophysics.
- These processes are thousands of light years away from us.
- Is it possible to distinguish such faraway fuzzy-type processes from more regular one?
20. The Main Difference Between Fuzzy-Type and Traditional Interactions

- Traditional interactions are smooth.
- In a small vicinity of each location, each smooth function can be well approximated by linear functions.
- Similarly, a curve can be approximated by its tangent.
- From this viewpoint, in a small vicinity, all smooth interactions are similar – they are all linear.
- This is where fuzzy-type interactions differ.
- The RHS $f(a, b) = \min(a, b)$ of the formula corr. to fuzzy-type interactions is not differentiable when $a = b$.
- We have $\frac{\partial f}{\partial a} = 1$ when $a < b$ and thus, $f(a, b) = a$.
- We have $\frac{\partial f}{\partial a} = 0$ when $a > b$ and thus, $f(a, b) = b$. 
21. We Cannot Use This Difference Directly

- Can we use this difference to directly distinguish non-smooth fuzzy-type interactions?
- Not really: even when we observe an Earth object from far away, the image is blurred (i.e., smoothed).
- In general, all remote signals are smoothed.
- So, irrespective of whether the original signal was smooth or not, the observed signal is smooth.
22. Let Us Try to Observe the Difference Indirectly

- While we cannot observe the non-smoothness directly, we may be able to observe it indirectly.

- Good news is that in physics, the rate of change – i.e., the derivative – of a quantity is also observable.

- For example, we can observe coordinates – and measure the corresponding location of an object.

- We can also directly measure the first derivative of the coordinate – the velocity – e.g., by its Doppler effect.

- We can even directly measure the second derivative of the coordinate – the acceleration – by an accelerometer.

- In our case, the first derivative \( \frac{dx}{dt} \) is described by a non-smooth function \( \min(a, b) \).
23. How to Observe the Difference

- Thus, the derivative of the first derivative – i.e., the acceleration \( \frac{d^2x}{dt^2} \) – is discontinuous.

- Discontinuity, however, will also be smoothed.

- Let us go one step further and take one more derivative, i.e., let us consider the third derivative \( \frac{d^3x}{dt^3} \).

- For a “jump” function, the derivative is infinite, so we will have an infinite third derivative.

- After smoothing, we will still have a huge value in the vicinity of the original non-smoothness.

- So, the question is: how can observe the third derivative?
24. How Can We Observe the Third Derivative: Analysis of the Problem

- In most real-life measurements, the first two derivatives dominate – to the extent that:
  - first two derivatives correspond to named physical quantities (velocity and acceleration),
  - there is no special word for the third derivative.
- To be able to observe the third derivative, we thus need to come up with a physical phenomenon:
  - in which the first two derivatives do not dominate,
  - i.e., when their effects are 0s (or at least small).
- In other words, we need a physical phenomenon in which accelerations do not count.
- If we have several bodies moving with the same acceleration, it is as if they are immobile.
25. How to Observe the 3rd Derivative (cont-d)

- There is a well-known phenomenon of this type.
- Namely, this is exactly what General Relativity and gravity are all about.
- Indeed, Einstein’s discovery of General Relativity started with his Equivalence Principle, according to which
  - a person in a freely falling elevator
  - will not notice any gravitation.
- In General Relativity, there is no absolute space or absolute time.
- What we measure when we measure spatial distances or time intervals depends on the bodies.
- Suppose that in some coordinate system, all the bodies move with the same acceleration $\vec{a}$. 

26. How to Observe the 3rd Derivative (cont-d)

- Then, we can change the coordinate system by taking the location of one of the bodies as the starting point.
- In the new coordinate system, all the bodies have 0 acceleration relative to each other.
- Thus, there is no acceleration at all.
- The situation is different if we have a change in acceleration – third derivative.
- It could be a change in time or change across space.
- This change cannot be eliminated by simply changing the coordinate system.
- Thus, our hope for detecting third derivatives lies in the analysis of the gravitation phenomena.
27. How Can We Find the Observable Consequences of Third Derivative in Gravitation?

- Gravitation force is very weak in comparison to all other physical forces.
- As a result, many phenomena are very difficult to observe for gravity – since they are very weak.
- Good news is that – at least on the Newtonian level:
  - the formulas for gravitation are similar to
  - the formulas for the electromagnetic field.
- In both cases, we have the same Coulomb’s law.
- The only difference is that similar electric charges repulse each other, while similar masses attract each other).
- Another good news is that electromagnetic forces are much stronger than the gravitational force.
28. 3rd Derivative in Gravitation (cont-d)

• This can be seen, e.g., by the fact that even small magnets easily overcome gravitation and pick up bodies.

• As a result of this difference in strength:
  – many phenomena which are difficult to observe and measure for gravity (since they are very weak)
  – can be easily observed and measured for electromagnetic processes.

• From this viewpoint:
  – to find the observable consequences of third derivative in gravitation,
  – let us recall observable effects of other derivatives in electromagnetic phenomena.
29. Electromagnetic Interactions: a Brief Reminder

- According to Coulomb’s laws, all electric changes interact with each other:
  - opposite charges attract each other, while
  - charges of different signs repel each other.

- Starting from Newton, interaction between the two bodies was understood as action-at-a-distance.

- So, a change in one object immediately affects all other objects in the Universe.

- In this description, interactions travel instantaneously – i.e., with infinite speed,

- However, according to relativity theory, no signal can propagate faster than the speed of light.

- No action-at-a-distance is possible.
30. Electromagnetic Interactions (cont-d)

- The only way a particle can affect another particle is via a field:
  - a particle changes the field in its vicinity,
  - this change leads to a change in the vicinity of this vicinity, etc.,
  - until the change reaches the location of the second particle.

- In quantum physics, everything is quantized, including the fields.

- Quanta of electromagnetic field are photons.

- Thus, interaction between two charged particles means, in effect, that:
  - one particle emits photons, and
  - photons then interact with other particles.
31. Electromagnetic Interactions (cont-d)

- In other words, particles exchange photons – and this exchange results in attraction or repulsion.
- In particular, different particles forming a changed body interact with each other by exchanging photons.
- When the body is inertial, this process of exchanging photons is stable.
- Some photons go in, some come out, everything is stable, no photons are lost, and no energy is lost.
- Indeed, otherwise, the loss of energy would mean that the body starts slowing down.
32. Electromagnetic Interactions (cont-d)

- In the coordinate system associated with its initial motion,
  - this would mean that the initially immobile body starts moving,
  - which contradicts to energy conservation law.

- However:
  - when a body deviates from the inertial trajectory
    - i.e., if there is an acceleration,
  - the balance between incoming and outgoing photos is disrupted.

- The number of photons going on corresponds to one velocity.

- The number of photons coming back in corresponds to a different velocity.
33. Electromagnetic Interactions (cont-d)

- As a result of this dis-balance, when a body accelerates, some photons are lost and some energy is lost.

- So, accelerated body emits some photons – i.e., *radiates*, emits what is called *electromagnetic waves*.

- The larger the acceleration, the larger the resulting photon flow.

- In the first approximation, the intensity of this photon flow (radiation) is proportional to the acceleration.
34. Back to Gravitational Interactions

- Gravity is also a field, its quanta are called *gravitons*.
- In contrast to the electromagnetic field, acceleration does not necessary means disbalance.
- It can be easily eliminated by changing a coordinate system.
- The only thing that cannot be eliminated by such a change is the third derivative.
- Thus, if there is a third derivative, there is a disbalance between outgoing and incoming gravitons.
- So, a body emits a *gravitational wave*.
- In the first approximation, the intensity of this wave
  - is proportional to the third derivative,
  - i.e., to the intensity of the fuzzy-type interactions.
35. What Type of Gravitational Waves Will We Observe?

- Gravitational waves are emitted when the third derivative is infinite.
- This corresponds to the case when the high concentrations are equal $a(t) = b(t)$.
- For a homogeneous body, in general, we have only one moment of time when this equality occurs.
- So, we will see a burst of gravitational waves.
- In non-homogeneous case, this above equality holds at different times at different locations.
- So, we will have an extended burst.
- By the duration of this burst, we can tell how non-homogeneous is the corresponding celestial body.
36. What Type of Gravitational Waves (cont-d)

- The expected bursts are different from the gravitational waves that we are observing now.

- These waves are *chirps*, periodic waves with a rapidly increasing frequency.

- These chirps is that they are caused by two bodies (e.g., two black holes) orbiting closely around each other.

- As they emit gravitational waves, they lose energy and thus, get closer and closer to each other.

- This, in accordance with the Kepler’s laws, causes the orbiting period to decrease.

- And thus, the frequency of the emitted waves increases.
37. Conclusions and Future Work

- In extremal conditions:
  - when the concentrations are very large,
  - some formulas describing physical interactions become fuzzy-type.

- The observable consequences of such fuzzy-type formulas is that they lead to bursts of gravitational waves.

- At present, our results are at a qualitative proof-of-concept level.

- It is desirable to raise these qualitative ideas to a more quantitative level.

- A possible way to do it is to perform numerical simulations.

- This will enable us to get numerical estimates of the corresponding physical phenomena.
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