Rotation-Invariance Can Further Improve State-of-the-Art Blind Deconvolution Techniques

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1. Image Deconvolution: Formulation of the Problem

- The measurement results $y_k$ differ from the actual values $x_k$ due to additive noise and blurring:

$$y_k = \sum_i h_i \cdot x_{k-i} + n_k.$$

- From the mathematical viewpoint, $y$ is a convolution of $h$ and $x$: $y = h \ast x$.

- Similarly, the observed image $y(i, j)$ differs from the ideal one $x(i, j)$ due to noise and blurring:

$$y(i, j) = \sum_{i'} \sum_{j'} h(i - i', j - j') \cdot x(i', j') + n(i, j).$$

- It is desirable to reconstruct the original signal or image, i.e., to perform deconvolution.
2. Ideal No-Noise Case

- In the ideal case, when noise \( n(i, j) \) can be ignored, we can find \( x(i, j) \) by solving a system of linear equations:

\[
y(i, j) = \sum_{i'} \sum_{j'} h(i - i', j - j') \cdot x(i', j').
\]

- However, already for 256×256 images, the matrix \( h \) is of size 65,536×65,536, with billions entries.

- Direct solution of such systems is not feasible.

- A more efficient idea is to use Fourier transforms, since \( y = h \ast x \) implies \( Y(\omega) = H(\omega) \cdot X(\omega) \); hence:

  - we compute \( Y(\omega) = \mathcal{F}(y) \);
  - we compute \( X(\omega) = \frac{Y(\omega)}{H(\omega)} \), and
  - finally, we compute \( x = \mathcal{F}^{-1}(X(\omega)) \).
3. Deconvolution in the Presence of Noise with Known Characteristics

- Suppose that signal and noise are independent, and we know the power spectral densities

\[ S_I(\omega) = \lim_{T \to \infty} E \left[ \frac{1}{T} \cdot |X_T(\omega)|^2 \right], \quad S_N(\omega) = \lim_{T \to \infty} E \left[ \frac{1}{T} \cdot |N_T(\omega)|^2 \right] \]

- We minimize the expected mean square difference

\[ d \overset{\text{def}}{=} \lim_{T \to \infty} \frac{1}{T} \cdot E \left[ \int_{-T/2}^{T/2} (\hat{x}(t) - x(t))^2 \, dt \right]. \]

- Minimizing \( d \) leads to the known Wiener filter formula

\[ \hat{X}(\omega_1, \omega_2) = \frac{H^*(\omega_1, \omega_2)}{|H(\omega_1, \omega_2)|^2 + \frac{S_N(\omega_1, \omega_2)}{S_I(\omega_1, \omega_2)}} \cdot Y(\omega_1, \omega_2). \]
4. Blind Image Deconvolution in the Presence of Prior Knowledge

- Wiener filter techniques assume that we know the blurring function $h$.
- In practice, we often only have partial information about $h$.
- Such situations are known as blind deconvolution.
- Sometimes, we know a joint probability distribution $p(\Omega, x, h, y)$ corresponding to some parameters $\Omega$:
  \[ p(\Omega, x, h, y) = p(\Omega) \cdot p(x|\Omega) \cdot p(h|\Omega) \cdot p(y|x, h, \Omega). \]
- In this case, we can find
  \[ \hat{\Omega} = \arg\max_{\Omega} p(\Omega|y) = \int \int_{x,h} p(\Omega, x, h, y) \, dx \, dh \]
  and
  \[ (\hat{x}, \hat{h}) = \arg\max_{x, h} p(x, h|\hat{\Omega}, y). \]
5. Blind Image Deconvolution in the Absence of Prior Knowledge: Sparsity-Based Techniques

- In many practical situations, we do not have prior knowledge about the blurring function $h$.
- Often, what helps is sparsity assumption: that in the expansion $x(t) = \sum_i a_i \cdot e_i(t)$, most $a_i$ are zero.
- In this case, it makes sense to look for a solution with the smallest value of
  \[ \|a\|_0 \overset{\text{def}}{=} \# \{ i : a_i \neq 0 \} . \]
- The function $\|a\|_0$ is not convex and thus, difficult to optimize.
- It is therefore replaced by a close convex objective function $\|a\|_1 \overset{\text{def}}{=} \sum_i |a_i|$.

- Sparsity is the main idea behind the algorithm described in (Amizic et al. 2013) that minimizes
  \[
  \frac{\beta}{2} \cdot \| y - W a \|^2_2 + \frac{\eta}{2} \cdot \| W a - H x \|^2_2 + \tau \cdot \| a \|_1 + \alpha \cdot R_1(x) + \gamma \cdot R_2(h).
  \]

- Here, \( R_1(x) = \sum_{d \in D} 2^{1-o(d)} \sum_i |\Delta^d_i(x)|^p \), where \( \Delta^d_i(x) \) is the difference operator, and

- \( R_2(h) = \| Ch \|^2 \), where \( C \) is the discrete Laplace operator.

- The \( \ell^p \)-sum \( \sum_i |v_i(x)|^p \) is optimized as \( \sum_i \frac{(v_i(x^{(k)}))^2}{v_i^{2-p}} \), where \( v_i = v_i(x^{(k-1)}) \) for \( x \) from the previous iteration.

- This method results in the best blind image deconvolution.
7. Need for Improvement

- The current technique is based on minimizing the sum $|\Delta_x I|^p + |\Delta_y I|^p$.

- This is a discrete analog of the term $\left| \frac{\partial I}{\partial x} \right|^p + \left| \frac{\partial I}{\partial y} \right|^p$.

- For $p = 2$, this is the square of the length of the gradient vector and is, thus, rotation-invariant.

- However, for $p \neq 2$, the above expression is not rotation-invariant.

- Thus, even if it works for some image, it may not work well if we rotate this image.

- To improve the quality of image deconvolution, it is thus desirable to make the method rotation-invariant.

- We show that this indeed improves the quality of deconvolution.
8. Rotation-Invariant Modification: Description and Results

- We want to replace the expression $\left| \frac{\partial I}{\partial x} \right|^p + \left| \frac{\partial I}{\partial y} \right|^p$ with a rotation-invariant function of the gradient.

- The only rotation-invariant characteristic of a vector $a$ is its length $\|a\| = \sqrt{\sum_i a_i^2}$.

- Thus, we replace the above expression with
  \[ \left( \left| \frac{\partial I}{\partial x} \right|^2 + \left| \frac{\partial I}{\partial y} \right|^2 \right)^{p/2}. \]

- Its discrete analog is $((\Delta_x I)^2 + (\Delta_y I)^2)^{p/2}$.

- This modification leads to a statistically significant improvement in reconstruction accuracy $\|\hat{x} - x\|_2$. 
9. Testing the New Algorithm: Details

• To test the new method, we compared it with the original methods:
  – on the same “Cameraman” image use in the original method,
  – with the same values of the parameters ($\alpha = 1$, $\gamma = 5 \cdot 10^5$, $\tau = 0.125$, $\eta^1 = 1024$);
  – we applied the same Gaussian blurring with the variance of 5;
  – with the same S/N ratio corr. to $\sigma = 0.001$.

• We used the same criterion $\|x - \hat{x}\|_2$ to gauge the deconvolution quality.

• Both methods start with randomly selected initial values $v^{1,1}_d$.

• Because of this, the results differ slightly when we reapply the algorithm to the same image.
10. Testing the New Algorithm (cont-d)

- Because of the statistical character of the results:
  - we apply both algorithms to the same image several times, and
  - we use statistical criteria to decide which method is better.

- To perform this comparison, we applied each of the two algorithms 30 times.

- To make the results more robust, we eliminated the smallest and the largest value of this distance.

- The averages of the remaining 28 distances are:
  - for the original algorithm 1195.21,
  - for the new algorithm, $1191.01 < 1195.21$. 
11. Testing the New Algorithm: Results

- To check whether this difference is statistically significance, we applied the t-test for two independent means:

\[ t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left(\frac{(N_1 - 1) \cdot s_1^2 + (N_2 - 1) \cdot s_2^2}{N_1 + N_2 - 2}\right) \cdot \left(\frac{1}{N_1} + \frac{1}{N_2}\right)}}. \]

- The null hypothesis is that both samples come from the populations with same mean.

- For the two above samples, computations lead to rejection with \( p = 0.002 \).

- This is much smaller than the \( p \)-values 0.01 and 0.05 normally used for rejecting the null hypothesis.

- Therefore, the modified algorithm is statistically significantly better than the original one.
12. Conclusions and Future Work

- Often, we need to reconstruct an image in situations when we do not know the blurring function.
- There exist empirically successful algorithms for such blind image deconvolution.
- While the current methods are reasonably efficient, they are not yet perfect; for example:
  - the current method correctly reconstructs the standard “Cameraman” image from its blurred version,
  - but when we rotated this image, the quality of the reconstruction drastically decreased.
- Making the first-order regularization terms rotation-invariant statistically significantly improves the image.
- It may be a good idea to try a similar replacement for second-order regularization terms.
13. Acknowledgments

This work was supported in part:

- by the National Science Foundation grants:
  - HRD-0734825 and HRD-1242122 (Cyber-ShARE Center of Excellence) and
  - DUE-0926721, and
- by an award from Prudential Foundation.