Soft Computing Approach to Detecting Discontinuities: Seismic Analysis and Beyond

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1. Formulation of the Problem

- Starting from Newton, the main equations of physics are differential equations.
- This fact implicitly implies that all the corresponding processes are differentiable – and thus, continuous.
- In practice, we often encounter processes or objects that change abruptly in time or in space. For example:
  - In physics, we have phase transitions when the properties change abruptly.
  - In geosciences, we have faults and we have sharp boundaries between different layers.
2. It Is Important to Detect Discontinuities

- In *civil engineering*, discontinuities may indicate crack or faults.
- Finding them can help us check structural integrity of the structure – e.g., of an airplane or a spaceship.
- In *geosciences*, faults are places where most earthquakes originate.
- So finding the exact locations of faults may help predict where earthquakes can happen in the future.
- In fracking, it is important to detect possible cracks.
- Through these cracks, chemicals in the pumped liquid can penetrate into the environment.
3. Detecting Discontinuities Is Often Not Easy

- In some cases, we know the corresponding equations.
- In such situations, we can use these equations to develop techniques for detecting discontinuities.
- In many other situations, however, we do not know the exact equations describing the process.
- For example:
  - while we may know that there is a fault,
  - we do not know the exact shape of this fault, and
  - we do not have a good understanding of how this fault interacts, e.g., with seismic waves.
- In such situations, all we know is the corresponding processes are discontinuous.
- How can we use this information to detect the discontinuities?
4. Need for Soft Computing

- At first glance, it looks like the word “discontinuity” has a precise mathematical meaning.

- However, this mathematical definition is not what the geophysicists have in mind.

- What they mean is rather a commonsense, informal meaning of this word.

- Our objective is thus to translate this imprecise meaning into a precise algorithm.

- In this translation, it is reasonable to use the technique of fuzzy logic.

- Indeed, this technique was designed to transform imprecise (“fuzzy”) expert knowledge into precise terms.
5. What Is Continuity?

- Continuity of \( a(t) \) means that if \( \Delta t \overset{\text{def}}{=} t' - t \) is small, then \( \Delta a \overset{\text{def}}{=} a(t') - a(t) \) should also be small.
- Let \( \mu(\Delta t) \) be the degree to which \( \Delta t \) is small.
- The quantity \( \Delta \) may use different units, so \( a \) corresponds to \( \lambda \cdot t \).
- So, \( \Delta t \) is equivalent to \( \Delta a/\lambda \), and smallness of \( \Delta a \) may be described as \( \mu(\Delta a/\lambda) \).
- “If \( A \) then \( B \)” means that if \( A \) is true, then \( B \) is true.
- Thus, the degree to which \( B \) is true should be greater than or equal to the degree to which \( A \) is true.
- In our case, this means \( \mu(\Delta a/\lambda) \geq \mu(\Delta t) \).
- The value \( \mu(\Delta t) \) decreases with \( \Delta t \), so \( \Delta a/\lambda \leq \Delta t \) and \( \left| \frac{\Delta a}{\Delta t} \right| \leq \lambda \).
6. So How Can We Detect Discontinuity?

- So, we can detect discontinuities by comparing a threshold $\lambda$ with the ratio

$$\left|\frac{\Delta a}{\Delta t}\right| = \left|\frac{a(t') - a(t)}{t' - t}\right|.$$ 

- As long as the ratio is below the threshold, we are continuous.

- Once the ratio is above the threshold, this is an indication of discontinuity.

- In some cases, the values $t$ are equally spaced:

$$t_1, \ t_2 = t_1 + \delta t, \ldots, t_k = t_{k-1} + \delta t, \ldots$$

- In such case, the desired ratio is simply proportional to $|a(t_k) - a(t_{k-1})|$.

- In this case, we get an even simpler criterion for detecting discontinuity.
7. How Can We Detect Discontinuity (cont-d)

- In general, a process is continuous if $|\Delta a/\Delta t| \leq \lambda$.
- We consider the case when the values $t$ are equally spaced: $t_1, t_2 = t_1 + \delta t, \ldots, t_k = t_{k-1} + \delta t, \ldots$
- Then, the ratio $|\Delta a/\Delta t|$ is proportional to $|a(t_k) - a(t_{k-1})|$.
- So, if the difference $|a(t_k) - a(t_{k-1})|$ does not exceed $\lambda \cdot \delta t$, then at the location $t_k$, the process is continuous.
- If the difference $|a(t_k) - a(t_{k-1})|$ exceeds this threshold, then at this location, there is a discontinuity.
- This conclusion is consistent with common sense – which is one more reason to trust it.
8. Application to Seismic Analysis

• In this paper, we used the experimental results from the 2014 Southern California study; in this study:
  – more than 1000 seismic sensors were placed on a dense 600 m × 600 m grid
  – on top of one of the known faults – San Jacinto fault.

• These sensors were in place for a 5-week period.

• As a result, we have the values $v(s, t)$ measured by different sensors $s$ at different moments of time $t$.

• During this period, the sensors recorded many earthquakes, both:
  – weak earthquakes originating in the vicinity of the fault and
  – stronger earthquake that occurred outside the fault.
Figure 1: Sensors placed in the vicinity of San Jacinto fault
9. What We Did

- Let $t_r(s, E)$ be moment when the signal from the earthquake $E$ reached sensor $s$.
- For each sensor $s$, we record the signal $v(s, t)$ for 10 seconds: $t_r(s, E) \leq t \leq t_r(s, E) + 10$.
- The signal interacts with the fault (and with other inhomogeneities).
- Thus, the shape of the signal at different sensors was somewhat different.
- As a measure of how the earthquake $E$ influenced the sensor $s$, we took:

$$m(s, E) = \max_{t_r(s, E) \leq t \leq t_r(s, E) + 10} |v(s, t)|.$$
10. What We Did

- For each earthquake $E$ and sensor $s$, we computed:
  \[ m(s, E) = \max_{t_r(s, E) \leq t \leq t_r(s, E) + 10} |v(s, t)|. \]

- Then, for each straight line of sensors $\ell$ in the direction of wave propagation:
  - we consider all the sensors $s_1(\ell), s_2(\ell), \ldots$ along this line;
  - for each of these sensors, we computed $m(s_k(\ell), E)$.

- Then, we computed the differences
  \[ |m(s_k(\ell), E) - m(s_{k-1}(\ell), E)|. \]
Figure 2: Wave field for P-wave across the fault
11. Results

- In almost all cases, the difference $|m(s_k(\ell), E) - m(s_{k-1}(\ell), E)|$ spiked when the line crossed the fault.

- For each line, we can identify the fault as the location at which the difference exceeds some threshold $\lambda$:

$$|m(s_k(\ell), E) - m(s_{k-1}(\ell), E)| \geq \lambda.$$ 

- This is in perfect accordance with the above soft-computing-based formula for $a(t_k) \overset{\text{def}}{=} m(s_k(\ell), E)$. 

12. Discussion

- Interestingly, the dependence of \( m(s_k(\ell), E) \) on \( k \) was drastically different for different earthquakes.

- For earthquake waves whose direction is \( \perp \) to the fault, \( m(s_k(\ell), E) \) increases when we cross the fault.

- For waves parallel to the fault, \( m(s_k(\ell), E) \) decreases when we cross the fault, then increases back.

- In all the cases, however, what was common was the fact that there was a drastic change around the fault.

- We can therefore use this change to detect the discontinuities.
13. Conclusions and Future Work

- In this paper:
  - on the example of detecting faults from seismic waves,
  - we show that methods based on soft-computing interpretation of discontinuity are helpful.
- We tested this method on the example of detecting the location of the San Jacinto fault.
- We hope that this success will enable us also to also detect difficult-to-detect cracks caused by fracking.
- Thus, it will help prevent possible ecological disasters.
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15. Appendix: Why Different Seismic Waves Behave Differently?

- We observed that:
  - when a seismic wave approaches the fault straight ahead, the amplitude increases;
  - when the earthquake wave approaches the fault at an angle closer to 0, the amplitude decreases.
- When the seismic wave hits the fault, part of its energy is diverted to directions close to orthogonal to the fault.
- As a result:
  - for waves whose direction is almost orthogonal to the fault, we measure a larger amplitude, while
  - for waves at a small angle to the fault, the energy decreases.
16. Scattering: Future Plans

- Such a phenomenon is well known in wave propagation as *scattering*:
  - when a wave approaches a point-wise obstacle, the scattered wave goes in all directions;
  - when a wave approaches a planar obstacle, we get scattered waves orthogonal to this obstacle.

- We hope that the known formulas of scattering seismic waves can help us go:
  - from the current idea of detecting the location of the fault
  - to a more detailed description of this fault.