Towards a Better Understanding of Space-Time Causality: Kolmogorov Complexity and Causality as a Matter of Degree

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1. Defining Causality Is Important

- Causal relation \( e \preceq e' \) between space-time events is one of the fundamental notions of physics.

- In Newton's physics, it was assumed that influences can propagate with an arbitrary speed:
  \[
  e = (t, x) \preceq e' = (t', x') \iff t \leq t'.
  \]

- In Special Relativity, the speeds of all the processes are limited by the speed of light \( c \):
  \[
  e = (t, x) \preceq e' = (t', x') \iff c \cdot (t' - t) \geq d(x, x').
  \]

- In the General Relativity Theory, the space-time is curved, so the causal relation \( \preceq \) is even more complex.

- Different theories, in general, make different predictions about the causality \( \preceq \).

- So, to experimentally verify fundamental physical theories, we need to experimentally check whether \( e \preceq e' \).
2. Defining Causality: Challenge

- Intuitively, \( e \preceq e' \) means that:
  - what we do in the vicinity of \( e \)
  - changes what we observe at \( e' \).
- If we have two (or more) copies of the Universe, then:
  - in one copy, we perform some action at \( e \), and
  - we do not perform this action in the second copy.
- If the resulting states differ, this would indicate \( e \preceq e' \):
  
<table>
<thead>
<tr>
<th>World 1</th>
<th>World 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>rain</td>
<td>rain dance</td>
</tr>
<tr>
<td>( e' )</td>
<td>no rain</td>
</tr>
<tr>
<td>( e )</td>
<td>no rain dance</td>
</tr>
</tbody>
</table>

- Alas, in reality, we only observe one Universe, in which we either perform the action or we do not.
3. Algorithmic Randomness and Kolmogorov Complexity: A Brief Reminder

- If we flip a coin 1000 times and still get all heads, common sense tells us that this coin is not fair.
- Similarly, if we repeatedly flip a fair coin, we cannot expect a periodic sequence 0101…01 (500 times).
- Traditional probability theory does not distinguish between random and non-random sequences.
- Kolmogorov, Solomonoff, Chaitin: a sequence 0…0 isn’t random since it can be printed by a short program.
- In contrast, the shortest way to print a truly random sequence is to actually print it bit-by-bit: \texttt{printf}(01…).
- Let an integer $C > 0$ be fixed. We say that a string $x$ is random if $K(x) \geq \text{len}(x) - C$, where
  \[
  K(x) \overset{\text{def}}{=} \min \{\text{len}(p) : p \text{ generates } x\}.
  \]
4. The Corresponding Notion of Independence

- If $y$ is independent on $x$, then knowing $x$ does not help us generate $y$.

- If $y$ depends on $x$, then knowing $x$ helps compute $y$; example:
  - knowing the locations and velocities $x$ of a mechanical system at time $t$
  - helps compute the locations and velocities $y$ at time $t + \Delta t$.

- Let an integer $C > 0$ be fixed. We say that a string $y$ is independent of $x$ if $K(y \mid x) \geq K(y) - C$, where
  \[ K(y \mid x) \overset{\text{def}}{=} \min\{\text{len}(p) : p(x) \text{ generates } y}\].

- We say that a string $y$ is dependent on the string $x$ if
  \[ K(y \mid x) < K(y) - C. \]
5. How to Define Space-Time Causality: First Seemingly Reasonable Idea

• At first glance, we can check whether \( e \preceq e' \) as follows:
  - First, we perform observations and measurements in the vicinity of the event \( e \), and get the results \( x \).
  - We also perform measurements and observations in the vicinity of the event \( e' \), and produce \( x' \).
  - If \( x' \) depends on \( x \), i.e., if \( K(x' \mid x) \ll K(x') \), then we claim that \( e \) can casually influence \( e' \).

• If \( e \preceq e' \), then indeed knowing what happened at \( e \) can help us predict what is happening at \( e' \).

• However, the inverse is not necessarily true.

• We may have \( x \approx x' \) because both \( e \) and \( e' \) are influenced by the same past event \( e'' \).

• Example: both \( e \) and \( e' \) receive the same signal from \( e'' \).
6. Towards a Working Definition of Causality

- According to modern physics, the Universe is quantum in nature; for microscopic measurements:
  - we cannot predict the exact measurement results,
  - we can only predict probabilities of different outcomes; the actual observations are truly random.
- For each space-time event $e$:
  - we can set up such a random-producing experiment in the small vicinity of $e$, and
  - generate a random sequence $r_e$.
- This random sequence $r_e$ can affect future results.
- So, if we know the random sequence $r_e$, it may help us predict future observations.
- Thus, if $e \ll e'$, then for some observations $x'$ performed in the small vicinity of $e'$, we have $K(x' | r_e) \ll K(x')$. 
7. Discussion and Resulting Definition

- **Reminder:** when \( e \preceq e' \), then the random sequence \( r_e \) can affect the measurement results at \( e' \):
  \[
  K(x' \mid r_e) \ll K(x').
  \]

- If \( e \not\preceq e' \), then the random sequence \( r_e \) cannot affect the measurement results at \( e' \):
  \[
  K(x' \mid r_e) \approx K(x').
  \]

- So, we arrive at the following semi-formal definition:
  - For a space-time event \( e \), let \( r_e \) denote a random sequence generated in the small vicinity of \( e \).
  - We say \( e \preceq e' \) if for some observations \( x' \) performed in the small vicinity of \( e' \), we have
    \[
    K(x' \mid r_e) \ll K(x').
    \]

- Our definition follows the ideas of causality as *mark transmission*, with the random sequence as a mark.
8. This Definition Is Consistent with Physical Intuition

- If $e \preceq e'$, then we can send all the bits of $r_e$ from $e$ to $e'$.
- The signal $x'$ received in the vicinity of $e'$ will thus be identical to $r_e$.
- Thus, generating $x'$ based on $r_e$ does not require any computations at all: $K(x' \mid r_e) = 0$.
- Since the sequence $x' = r_e$ is random, we have $K(x') \geq \text{len}(x') - C$.
- When $r_e = x'$ is sufficiently long ($\text{len}(x') > 2C$), we have $K(x') \geq \text{len}(x') - C > 2C - C = C$, hence $0 = K(x' \mid r_e) < K(x') - C$ and $K(x' \mid r_e) \ll K(x')$.
- So, our definition is indeed in accordance with the physical intuition.
9. Randomness Is a Matter of Degree

- **Reminder**: a sequence $x$ is random if $K(x) \geq \text{len}(x) - C$ for some $C > 0$.

- For a given sequence $x$, its **degree of randomness** $d(x)$ can be defined by the smallest integer $C$ for which

  $$K(x) \geq \text{len}(x) - C.$$ 

- One can check that this smallest integer is equal to the difference $d(x) = \text{len}(x) - K(x)$.

- For *random* sequences, the degree $d(x)$ is small.

- For sequences which are *not random*, the degree $d(x)$ is large.

- In general, the smaller the difference $d(x)$, the more random is the sequence $x$. 
10. Space-Time Causality Is a Matter of Degree

- Our definition of causality is that $K(x' | r_e) < K(x') - C$ for some large integer $C$.
- The larger the integer $C$, the more confident we are that an event $e$ can causally influence $e'$.
- It is therefore reasonable to define a degree of causality $c(e, e')$ as the largest integer $C$ for which
  \[ K(x' | r_e) < K(x') - C. \]
- One can check that this largest integer is equal to the difference $c(e, e') = K(x') - K(x' | r_e) - 1$.
- The larger this difference $c(e, e')$, the more confident we are that $e$ can influence $e'$.
- In other words, just like randomness turns out to be a matter of degree, causality is also a matter of degree.
11. Remaining Open Problems

- It is desirable to explore possible physical meaning of such “degrees of causality” $c(e, e')$.

- Maybe this function $c(e, e')$ is related to relativistic metric – the amount of proper time between $e$ and $e'$?

- Another open problem: the above definition works for objects in a small vicinity of one spatial location.

- In quantum physics, not all objects are localized in space-time.

- We can have situations when the states of two spatially separated particles are entangled.

- It is desirable to extend our definition to such objects as well.
12. Conclusions

- We propose a new operationalist definition of causality \( e \preceq e' \) between space-time events \( e \) and \( e' \).
- Namely, to check whether an event \( e \) can casually influence an event \( e' \), we:
  - generate a truly random sequence \( r_e \) in the small vicinity of the event \( e \), and
  - perform observations in the small vicinity of the event \( e' \).
- If some observation results \( x' \) (obtained near \( e' \)) depend on \( r_e \), then we claim that \( e \preceq e' \).
- On the other hand, if all observation results \( x' \) are independent on \( r_e \), then we claim that \( e \npreceq e' \).
- This new definition naturally leads to a conclusion that space-time causality is a matter of degree.
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