Assessment of Functional Impairment in Human Locomotion: Fuzzy-Motivated Approach

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1. Formulation of the Problem

- Neurological disorders – e.g., the effects of a stroke – affect human locomotion (such as walking).

- In most cases, the effect of a neurological disorder can be mitigated by applying an appropriate rehabilitation.

- For the rehabilitation to be effective, it is necessary to be able:
  - to correctly diagnose the problem,
  - to assess its severity, and
  - to monitor the effect of rehabilitation.

- At present, this is mainly done subjectively, by experts who observe the patient.

- This is OK for the diagnosis, but for rehabilitation, a specialist can see a patient only so often.
2. Formulation of the Problem (cont-d)

- It is desirable to *automatically* gauge how well the patient progresses.

- To measure the gait \( x(t) \), we can use:
  - inertial sensors that measure the absolute and relative location of different parts of the body, and
  - electromyograph (EMG) sensors that measure the electric muscle activity during the motion.

- By comparing \( x(t) \) with gait of healthy people and with previous patient’s gait, we can:
  - gauge how severe is the gait disorder, and
  - gauge whether the rehabilitation is helping.

- *Problem:* signals \( x(t) \) corresponding to patients and to healthy people are similar.
3. Need for Fuzzy Techniques

- Specialists can distinguish between signals corr. to patients and healthy people.
- We want to automate this specialists’ skill.
- Specialists describe their decisions by using imprecise (“fuzzy”) words from natural language.
- Formalizing such words is one of the main tasks for which fuzzy techniques have been invented.
- Fuzzy techniques have been used to design efficient semi-heuristic assessment systems.
- The objective of this paper is to provide a theoretical justification for the existing fuzzy systems.
- The existence of such a justification makes the results of the system more reliable.
4. Pre-Processing of Gait Signal

- Motions differ by speed: the same person can walk slower or faster.

- To reduce the effect of different speeds, we re-scale time
  \[ x'(T) = x(t_0 + T \cdot T_0), \]
  where
  \- \( t_0 \) is the beginning of the gait cycle,
  \- \( T_0 \) is the gain cycle, and
  \- the new variable \( T \) describe the position of the sensor reading on the gait cycle.

- For example:
  \- the value \( x'(0) \) describes the sensor’s reading at the beginning of the gait cycle,
  \- the value \( x'(0.5) \) describes the sensor’s reading in the middle of the gait cycle,
  \- the value \( x'(0.25) \) describes the sensor’s reading at the quarter of the gait cycle.
5. Pre-Processing of Gait Signal (cont-d)

- Motions also differ by intensity.
- To reduce the effect of different intensities, we re-scale the signal $x(t)$ so that:
  - the smallest value on each cycle is 0, and
  - the largest value on each cycle is 1.
- Such a scaling has the form $X(T) = \frac{x'(T) - x}{\overline{x} - \underline{x}}$, where:
  - $\underline{x}$ is the smallest possible value of the signal $x'(T)$ during the cycle, and
  - $\overline{x}$ is the largest possible value during the cycle.
- After re-scaling, all we have to do is compare:
  - the (re-scaled) observed signal $X(T)$ with
  - a similarly re-scaled signal $X_0(T)$ corresponding to the average of normal behaviors.
6. Fuzzy Gait Assessment System

- An expert describes the gait by specifying how the motion looked like at different \( p \) parts of the gait cycle.
- For each part, we form a triangular membership function \( \mu(x) \) that best describes the corr. values \( X(T) \).
- We want the support \((a, b)\) of \( \mu(x) \) to be narrow and to contain many observed values \( x_i \).
- Pedrycz’s approach: find parameters \( a, b, m \) for which
  \[
  \sum_{i=1}^{n} \frac{\mu(x_i)}{b - a} \to \text{max}.
  \]
- The gait on each part is described by three parameters \((a, b, m)\), so overall we need \( N = v3p \) parameters.
- A patient’s gait is described by \( g_1, \ldots, g_N \in [0, 1] \).
- The normal gait is described by \( n_1, \ldots, n_N \in [0, 1] \).
7. Fuzzy Gait Assessment System (cont-d) and Our Result

- A sequence $g_1, \ldots, g_N$ can be viewed as a fuzzy set $g$.
- A sequence $n_1, \ldots, n_N$ can be viewed as a fuzzy set $n$.
- So, we can define degree of similarity between patient’s gait and normal gait as

$$s = \frac{|g \cap n|}{|g \cup n|} = \frac{\sum_{i=1}^{N} \min(g_i, n_i)}{\sum_{i=1}^{N} \min(g_i, n_i)}.$$ 

- Our result: when the number of parts $p$ is large enough, we have

$$s \approx 1 - \frac{1}{C} \cdot \int |x(t) - x_0(t)| \, dt.$$ 

- Thus, the larger the integral, the more severe the disorder.
8. Explanation of the Reformulated Formula

- Let’s explain why \( \int |\Delta x(t)| \, dt \), where \( \Delta x(t) \overset{\text{def}}{=} x(t) - x_0(t) \), is a good measure of the disorder’s severity.

- The effect is different for different behaviors.

- It is reasonable to gauge the severity of a disorder by the worst-case effect of this difference.

- For each objective, the effectiveness \( E \) of this activity depends on the differences \( \Delta x(t_i) \).

- The differences \( \Delta x(t_i) \) are small, so we can linearize the dependence: \( \Delta E = \sum c_i \cdot \Delta x(t_i) \).

- There is a bound \( M \) on possible values of \( |c_i| \).

- The largest value of \( \sum c_i \cdot \Delta x(t_i) \) under the constraint \( |c_i| \leq M \) is equal to \( M \cdot \sum |\Delta x(t_i)| \).

- Thus, the worst-case effect is indeed proportional to \( \sum |\Delta x(t_i)| \), i.e., to \( \int |\Delta x(t)| \, dt \).
9. Conclusion and Future Work

- Many traumas and illnesses result in motion disorders.
- In many cases, the effects of these disorders can be decreased by an appropriate rehabilitation.
- Different patients react differently to the current rehabilitation techniques.
- To select an appropriate technique, it is therefore extremely important to be able to gauge:
  - how severe is the current disorder and
  - how much progress has been made in the process of rehabilitation.
- At present, this is mostly done subjectively, by a medical doctor periodically observing the patient’s motion.
- When a certain therapy does not help, the doctor can change the rehabilitation procedure.
10. Conclusion and Future Work (cont-d)

• It is desirable to make frequent evaluations, to make sure that the procedure indeed improves the patient.

• For that, it is desirable to come up with ways to automatically access the patient’s progress.

• In previous papers, fuzzy techniques were used to design semi-heuristic assessment techniques.

• In this paper, we provide a theoretical justification for these techniques.

• In the future, it is desirable:
  – to enhance these fuzzy-based assessment techniques
  – by combining them with fuzzy-based techniques for modeling gait (and other motions).
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12. Proof of the Main Result

- On each part \( i \), the motion changes slightly from the midpoint \( x(t_i) \): all observed values \( x(t) \) are \( x(t) \approx x(t_i) \).

- Hence, the values \( a, b, \) and \( m \) are also close to \( x(t_i) \), so

\[
\begin{align*}
3 \cdot \sum_{i=1}^{n} \min(x(t_i), x_0(t_i)) &= \sum_{i=1}^{n} \min(x(t_i), x_0(t_i)) \\
3 \cdot \sum_{i=1}^{n} \max(x(t_i), x_0(t_i)) &= \sum_{i=1}^{n} \max(x(t_i), x_0(t_i))
\end{align*}
\]

- Since \( \Delta x(t) = x(t) - x_0(t) \), we get \( x(t) = x_0(t) + \Delta x(t) \), and:

\[
s = \frac{\sum_{i=1}^{n} \min(x_0(t_i) + \Delta x(t_i), x_0(t_i))}{\sum_{i=1}^{n} \max(x_0(t_i) + \Delta x(t_i), x_0(t_i))}.
\]
13. Proof (cont-d)

\[ s = \frac{\sum_{i=1}^{n} \min(x_0(t_i) + \Delta x(t_i), x_0(t_i))}{\sum_{i=1}^{n} \max(x_0(t_i) + \Delta x(t_i), x_0(t_i))}. \]

- Reminder: \( s = \frac{\sum_{i=1}^{n} \min(x_0(t_i) + \Delta x(t_i), x_0(t_i))}{\sum_{i=1}^{n} \max(x_0(t_i) + \Delta x(t_i), x_0(t_i))}. \)

- Here, if \( \Delta x(t_i) \geq 0 \), then
  \[ \min(x_0(t_i) + \Delta x(t_i), x_0(t_i)) = x_0(t_i). \]

- If \( \Delta x(t_i) < 0 \), then
  \[ \min(x_0(t_i) + \Delta x(t_i), x_0(t_i)) = x_0(t_i) + \Delta x(t_i). \]

- Similar formulas hold for max, so for \( s_0 \overset{\text{def}}{=} \sum_{i=1}^{n} x_0(t_i) \), we get
  \[ s = \frac{s_0 + \sum_{i: \Delta x(t_i) < 0} \Delta x(t_i)}{s_0 + \sum_{i: \Delta x(t_i) \geq 0} \Delta x(t_i)}. \]
14. Proof (cont-d)

- Dividing both the numerator and the denominator by $s_0$, we conclude that

$$s = \frac{s_0 + \sum_{i: \Delta x(t_i) < 0} \Delta x(t_i)}{s_0 + \sum_{i: \Delta x(t_i) \geq 0} \Delta x(t_i)} = \frac{1 + \sum_{i: \Delta x(t_i) < 0} \frac{\Delta x(t_i)}{s_0}}{1 + \sum_{i: \Delta x(t_i) \geq 0} \frac{\Delta x(t_i)}{s_0}}.$$

- Since $|\Delta x(t_i)| \ll x(t_i)$, we can use the fact that

$$\frac{1 + a}{1 + b} \approx (1 + a) \cdot (1 - b + \ldots) = 1 + a - b + \ldots$$

- Thus, $s \approx 1 + \sum_{i: \Delta x(t_i) < 0} \frac{\Delta x(t_i)}{s_0} - \sum_{i: \Delta x(t_i) \geq 0} \frac{\Delta x_0(t_i)}{s_0}$.

- Hence $s = 1 + \frac{1}{s_0} \cdot \left( \sum_{i: \Delta x(t_i) < 0} \Delta x(t_i) - \sum_{i: \Delta x(t_i) \geq 0} \Delta x(t_i) \right)$.
15. Proof (final part)

- Reminder: \( s = 1 + \frac{1}{s_0} \cdot \left( \sum_{i: \Delta x(t_i) < 0} \Delta x(t_i) - \sum_{i: \Delta x(t_i) \geq 0} \Delta x(t_i) \right) \).

- So, \( s \approx 1 - \frac{1}{s_0} \cdot \sum_{i=1}^{n} |\Delta x(t_i)|. \)

- Once we multiply this sum by \( \Delta t = t_{i+1} - t_i \), we get an integral sum \( \sum_{i=1}^{n} |\Delta x(t_i)| \cdot \Delta t \) for the interval \( \int |\Delta x(t)| \, dt. \)

- So, the dissimilarity (i.e., the severity of the disorder) is proportional to the integral \( I \overset{\text{def}}{=} \int |\Delta x(t)| \, dt. \)