How to Estimate, Take Into Account, and Improve Travel Time Reliability in Transportation Networks

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1. Decreasing Traffic Congestion: Formulation of the Problem

- **Practical problem:** decreasing traffic congestion.
- **Important difficulty:** a new road can worsen traffic congestion.
- **Conclusion:** importance of the preliminary analysis of the results of road expansion.
- **Traditional approach** assumes that we know:
  - the exact amount of traffic going from zone A to zone B (*OD*-matrix), and
  - the exact capacity of each road segment.
- **Limitations:** in practice, we only know all this with uncertainty.
- **What we do:** we show how to take this uncertainty into account in traffic simulations.
2. Traffic Assignment: Brief Reminder

- **Traffic demand:** \# of drivers \( d_{ij} \) who need to go from zone \( i \) to zone \( j \) – *origin-to-destination* (O-D) matrix.

- **Capacity** of a road link – the number \( c \) of cars per hour which can pass through this link.

- **Travel time along a link:** \( t = t^f \cdot \left[ 1 + a \cdot \left( \frac{v}{c} \right)^\beta \right] \), where:
  - \( t^f = L/s \) is a *free-flow* time (\( s \) is the speed limit),
  - \( a \approx 0.15 \) and \( \beta \approx 4 \) are empirical constants.

- **Equilibrium:** when
  - the travel time along all used alternative routes is exactly the same, and
  - the travel times along other un-used routes is higher.

- **Algorithms:** there exist efficient algorithms for finding the equilibrium.
3. How We Can Use the Existing Traffic Assignment Algorithms to Solve Our Problem: Analysis

- **Main objective:** predict how different road project affect future traffic congestion.

- **Future traffic demands – what is known:** there exist techniques for predicting daily O-D matrices.

- **What is lacking:** we need to “decompose” the daily O-D matrix into 1 hour (or 15 minute) intervals.

- **1st approximation:** assume that the proportion of drivers starting at, say 6 to 7 am is the same as now.

- **Need for a more accurate approximation:**
  - drivers may start early because of congestion;
  - if a new road is built, they will start later;
  - the % of those who start 6–7 am will decrease.

- **We cover:** both approximations.
4. Towards a More Accurate Approximation to O-D Matrices

- **Describing preferences**: empirical utility formula
  \[
  u_i = -0.1051 \cdot E(T) - 0.0931 \cdot E(SDE) - 0.1299 \cdot E(SDL) - 1.3466 \cdot P_L - 0.3463 \cdot \frac{S}{E(T)},
  \]
  where \( E(X) \) means expected value,
  - \( T \) is the travel time \( T \),
  - \( SDE \) is the wait time when arriving early,
  - \( SDL \) is the delay when arriving late,
  - \( P_L \) is the probability of arriving late, and
  - \( S \) is the variance of the travel time.

- **Logit model**: the probability \( P_i \) that a driver will choose the \( i \)-th time interval is proportional to \( \exp(u_i) \):
  \[
  P_i = \frac{\exp(u_i)}{\exp(u_1) + \ldots + \exp(u_n)}.
  \]
5. A Seemingly Natural Idea and Its Limitations

- **Seemingly natural idea:**
  - start with the 1st approximation O-D matrices $M_1$;
  - based on $M_1$, we find travel times, and use them to find the new O-D matrices $M_2 \equiv F(M_1)$;
  - based on $M_2$, we find travel times, and use them to find the new O-D matrices $M_3 \equiv F(M_2)$;
  - repeat until converges.

- **Toy example illustrating a problem:**
  - now: no congestion, all start at 7:30, work at 8 am;
  - $M_1$: full O-D matrix for 7:30 am, 0 for 7:15 am;
  - based on this $M_1$, we get huge delays;
  - $M_2$: everyone leaves for work early at 7:15 am;
  - at 7:30, roads are freer, so in $M_3$, all start at 7:30;
  - no convergence: $M_1 = M_3 = \ldots \neq M_2 = M_4 \ldots$
6. A More Realistic Approach

- *Above idea:* drivers make decisions based only on previous day traffic.

- *More accurate idea:* drivers make decisions based on the average traffic over a few past days.

- *Resulting process:*
  - start with the 1st approximation O-D matrices $M_1$;
  - for $i = 2, 3, \ldots$:
    * compute the average $E_i = \frac{M_1 + \ldots + M_i}{i}$,
    * find traffic times based on $E_i$;
    * use these traffic times to compute a new O-D matrix $M_{i+1} = F(E_i)$;
    * repeat until converges.

- *Process converges:* on toy examples, on El Paso network, etc.
7. Algorithm Simplified

- **Main idea:** once we know the previous average $E_i$, we can compute

$$E_{i+1} = \frac{(M_1 + \ldots + M_i) + M_{i+1}}{i + 1} = \frac{i \cdot E_i + M_{i+1}}{i + 1} = E_i \cdot \left(1 - \frac{1}{i+1}\right) + M_{i+1} \cdot \frac{1}{i + 1}.$$ 

- **We know:** that $M_{i+1} = F(E_i)$.

- **Resulting algorithm:**
  - start with the 1st approximation O-D matrices

$$E_1 = M_1;$$

  - compute $E_{i+1} = E_i \cdot \left(1 - \frac{1}{i+1}\right) + F(E_i) \cdot \frac{1}{i + 1};$

  - repeat until converges.
8. Taking Uncertainty into Account

- **Deterministic model:** \( t = t^f \cdot \left[ 1 + a \cdot \left( \frac{v}{c} \right)^\beta \right] \).

- **Traffic assignment:** a driver minimizes the travel time \( t = t_1 + \ldots + t_n \).

- **In practice:** travel times vary.

- **Decision theory:** maximize expected utility \( E[u] \).

- **How utility depends on travel time:** \( u(t) = -U(t) \), where \( U(t) = \exp(\alpha \cdot t) \).

- **Conclusion:** the driver minimizes
  \[
  E[U(t)] = E[\exp(\alpha \cdot t)] = E[\exp(\alpha \cdot (t_1 + \ldots + t_n))] = E[\exp(\alpha \cdot t_1) \cdot \ldots \cdot \exp(\alpha \cdot t_n)].
  \]

- Deviations on different links are independent, so
  \[
  E[U(t)] = E[\exp(\alpha \cdot t_1)] \cdot \ldots \cdot E[\exp(\alpha \cdot t_n)].
  \]
9. Taking Uncertainty into Account (cont-d)

- Minimizing $E[U(t)] = E[\exp(\alpha \cdot t_1)] \cdot \ldots \cdot E[\exp(\alpha \cdot t_n)]$\
  $\Leftrightarrow$ minimizing $\sum_{i=1}^{n} \tilde{t}_i$, where $\tilde{t}_i \overset{\text{def}}{=} \ln(E[\exp(\alpha \cdot t_i)])$.

- $\tilde{t}$ depends on $t^f$ and $r \overset{\text{def}}{=} \frac{t - t^f}{t}$: $\tilde{t} = F(t^f, r)$.

- If we divide a link into sublinks, we conclude that $F(t^f_1 + t^f_2, r) = F(t^f_1, r) + F(t^f_2, r)$, hence $\tilde{t} = t^f \cdot k(r)$.

- For no-congestion case $r = 0$, we have $\tilde{t} = t^f$, so $k(0) = 1$ and $k(r) = 1 + a_0 \cdot r + a_2 \cdot r^2 + \ldots$

- Empirical analysis: $a_1 \approx 1.4, b \approx 0$, so\
  $$\tilde{t} = t^f \cdot \left[1 + a \cdot a_1 \cdot \left(\frac{v}{c}\right)^\beta\right].$$

- Solution: use the standard travel time formula with $a \cdot a_1 \approx 0.21$ instead of $a \approx 0.14$. 
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11. Logit Discrete Choice Model: A New Justification

- **Reasonable assumption:** if we add same incentive to all routes, probabilities will not change.
- **For 2 routes:** $P_1 = F(\Delta V)$, where $\Delta V \equiv V_1 - V_2$.
- **Bayes theorem:**
  
  $$P(H_i \mid E) = \frac{P(E \mid H_i) \cdot P_0(H_i)}{P(E \mid H_1) \cdot P_0(H_1) + \cdots + P(E \mid H_n) \cdot P_0(H_n)}.$$  

- **Idea:** if we add an incentive $v_0$ to one of the routes, this changes the probability of selecting this route:
  
  $$F(\Delta V + v_0) = \frac{A(v_0) \cdot F(\Delta V)}{A(v_0) \cdot F(\Delta V) + B(v_0) \cdot (1 - F(\Delta V))}.$$  

- **Conclusion:** $F(\Delta V) = \frac{1}{1 + e^{-\beta \cdot \Delta V}}$, so
  
  $$p_1 = F(V_1 - V_2) = \frac{e^{\beta \cdot V_1}}{e^{\beta \cdot V_1} + e^{\beta \cdot V_2}}.$$
12. **Towards an Optimal Algorithm for Computing Fixed Points**

- **Idea:** when iterations $x_{k+1} = f(x_k)$ do not converge,
  \[
x_{k+1} = x_k + \alpha \cdot (f(x_k) - x_k) = (1 - \alpha_k) \cdot x_k + \alpha_k \cdot f(x_k).
  \]

- **Question:** which choice of $\alpha_k$ is best?

- **Idea:** this is a discrete approximation to a continuous-time system
  \[
  \frac{dx}{dt} = \alpha(t) \cdot (f(x) - x).
  \]

- **Scale invariance:** the system should not change if we use a different discretization, i.e., re-scale $t$ to $t' = t/\lambda$:
  \[
  \frac{dx}{dt'} = (\lambda \cdot \alpha(\lambda \cdot t')) \cdot (f(x) - x).
  \]

- **Conclusion:** $\lambda \cdot \alpha(\lambda \cdot t') = a(t')$, so for $\lambda = 1/t'$, we get
  \[
  \alpha(t') = \frac{c}{t'}
  \]
  for some $c$.

- **Fact:** this is exactly what we used: $\alpha_k = 1/k$. 
13. Exponential Disutility Functions in Transportation Modeling: Justification

- **Situation:**

```
   t0       t1
  C --- A --- B
     t2
```

- **Reasonable assumption:** the driver starting at C will choose the same road as the driver starting at A.

- **Formally:** if $E[u(t_1)] < E[u(t_2)]$ then
  $E[u(t_1 + t_0)] < E[u(t_2 + t_0)]$.

- **Result:** $u(t) = t$, $u(t) = \exp(c \cdot t)$, or
  $u(t) = -\exp(-c \cdot t)$.

- **Fact:** this is exactly the empirically justified formula used in transportation.