Possibility of Objective Interval Uncertainty in Physics: Analysis

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1. Predictions in Newtonian Physics

- *In Newtonian physics*:
  - once we know the current state of the system,
  - we can predict (at least in principle) all the future states of this system.
- *In real life:*
  - measurements are never absolutely accurate, so we do not have the exact knowledge of the current state.
- *However:*
  - the more accurate our measurements of the current state, the more accurate predictions we can make.
- *The inaccuracy*
  - of the existing knowledge and
  - of the resulting predictions
- can often be described in terms of interval uncertainty.

2. Predictions in Quantum Physics

- *In quantum physics:*
  - we cannot predict the exact future state of a system;
  - we can only predict the probabilities of different future states.
- *According to the modern quantum physics:*
  - if we know the exact initial state of the world we can uniquely predict these probabilities.
- *This means:*
  - the more accurate our measurements of the current state, the more accurate predictions of probabilities we can make.
- *In practice:*
  - we can often predict the intervals of possible values of the probability.

3. Possible of Objective Interval-Valued Probabilities

- *It is reasonable to conjecture that:*
  - for some real-life processes,
  - there is no objective probability.
- *In other words:*
  - for different subsequences,
  - the corresponding frequencies can indeed take different values from a given interval.
- The analysis of such processes is given by Gorban in 2007.
- How can we go beyond frequencies in this analysis?
- *A common sense idea:*
  - if an event has probability 0,
  - then it cannot happen.
- This cannot be literally true since every number has probability 0, and thus, no number is random.

4. Kolmorogov's Definition of Randomness

- *A common sense idea* (reminder):
  - if an event has probability 0,
  - then it cannot happen.
- *Problem:* this cannot be literally true.
- *Reason:*
  - every number has probability 0, and
  - thus, no number is random.
- *Idea of Kolmorogov and Martin-Löf:* we only require that *definable* events of probability 0 do not happen.
- *Good news:* we get a consistent definition of randomness.
- *Reason:*
  - there are only countably many defining texts;
  - thus countably many definable events,
  - the union of countably many events of probability 0 has probability 0;
  - thus, we indeed have a consistent definition of a random object.

5. How Can We Define When an Object is Random

- *Randomness under a known probability distribution P* (reminder):
  - an object x is random
  - if its does not belong to any definable event E with $P(E) = 0$.
- *Meaning:* if a (definable) event E has probability 0, then it cannot happen.
- *New situation:*
  - we do not know the probability distribution;
  - we only know a class $\mathcal{P}$ of possible probability distributions.
- *Idea:* if a definable event E is guaranteed to have probability 0 (i.e., $P(E) = 0$ for all possible P) then it cannot happen.
- *Resulting definition:*
  - an object x is random
  - if it does not belong to any definable event E for which $P(E) = 0$ for all $P \in \mathcal{P}$.

6. Observation and a Surprising Result

- *Observation:*
  - if an object x is random w.r.t. some $P_0 \in \mathcal{P}$,
  - then it is also random w.r.t. $\mathcal{P}$.
- *Proof:*
  - let E be a definable event for which $P(E) = 0$ for all $P \in \mathcal{P}$;
  - we want to prove that $x \notin E$;
  - since $P(E) = 0$ for all $P \in \mathcal{P}$ and $P_0 \in \mathcal{P}$, in particular, $P_0(E) = 0$;
  - since x is P_0-random, we have $x \notin E$;
  - the observation is proven.
- *Case:* the class $\mathcal{P}$ is finite: $\mathcal{P} = \{P_1, \ldots, P_n\}$.
- *According to observation:* for every i, every P_i-random object is $\mathcal{P}$-random.
- *Natural expectation:* there are $\mathcal{P}$-random objects which are not P_i-random.
- *Surprising result:* every $\mathcal{P}$-random object is random with respect to one of the probability measures P_i.

7. Proof

- *Formulation of the result* (reminder): every $\mathcal{P}$-random object is random with respect to one of the probability measures P_i.
- *Proof:* by contradiction:
  - let x be $\mathcal{P}$-random and not random with respect to all P_i;
  - by definition, P_i-random means that $x \notin E$ for all definable E with $P_i(E) = 0$;
  - thus, the fact that x is not P_i-random means that there exists an event E_i with $P_i(E_i) = 0$ for which $x \in E_i$;
  - since $x \in E_i$ for all i, the object x belongs to the intersection $E \stackrel{\text{def}}{=} \bigcap\limits_{i=1}^{n} E_i$: $x \in E$;
  - since $P_i(E_i) = 0$ and $E \subseteq E_i$, we have $P_i(E) = 0$;
  - thus, x belongs to the event E for which $P_i(E) = 0$ for all i;
  - this contradicts to our assumption that x is $\mathcal{P}$-random;
  - the statement is proven.
- *We hope:* that this problem does not appear in the more physical interval-valued class $\mathcal{P}$.

8. References

(1) Gorban, I.I.; *Theory of Hyper-Random Phenomena*, Kyiv, Ukrainian National Academy of Sciences Publ, 2007 (in Russian).

- *Comment:* this book promotes the idea of objective interval-valued probabilities.

(2) Li, M., and Vitányi, P.: *An Introduction to Kolmogorov Complexity and Its Applications*, Springer, Berlin-Heidelberg, 1997.

- *Comment:* this book provides a general introduction to Kolmogorov complexity and randomness.

(3) Kreinovich, V., and Longpré, L.; International Journal on Theoretical Physics, 1997, Vol. 36, No. 1, pp. 167–176.

- *Comment:* this paper contains the ideas that we used in our proof.