A Simple Geometric Model Provides a Possible Quantitative Explanation of the Advantages of Immediate Feedback in Student Learning

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1. **Student Understanding is Extremely Important**

- One of the main objectives of a course – calculus, physics, etc. – is to help students understand its main concepts.
- Of course, it is also desirable that the students learn the corresponding methods and algorithms.
- However, understanding is the primary goal.
- If a student does not remember a formula by heart, she can look it up.
- However:
  - if a student does not have a good understanding of what, for example, is a derivative,
  - then even if this student remembers some formulas, he will not be able to decide which formula to apply.
2. How to Gauge Student Understanding

- To properly gauge student’s understanding, several disciplines have developed *concept inventories*.
- These are sets of important basic concepts and questions testing the students’ understanding.
- The first such Force Concept Inventory (FCI) was developed to gauge the students’ understanding of forces.
- A student’s degree of understanding is measured by the percentage of the questions that are answered correctly.
- The class’s degree of understanding is measured by averaging the students’ degrees.
- An ideal situation is when everyone has a perfect 100% understanding; in this case, the average score is 100%.
- In practice, the average score is smaller than 100%.
3. How to Compare Different Teaching Techniques

- We can measure the average score $\mu_0$ before the class and the average score $\mu_f$ after the class.
- Ideally, the whole difference $100 - \mu_0$ disappears, i.e., the students’ score goes from $\mu_0$ to $\mu_f = 100$.
- In practice, of course, the students’ gain $\mu_f - \mu_0$ is somewhat smaller than the ideal gain $100 - \mu_0$.
- It is reasonable to measure the success of a teaching method by which portion of the ideal gain is covered:

$$g \overset{\text{def}}{=} \frac{\mu_f - \mu_0}{100 - \mu_0}.$$
4. Empirical Results

• It turns out that the gain $g$ does not depend on the initial level $\mu_0$, on the textbook used, or on the teacher.

• Only one factor determines the value $g$: the absence or presence of immediate feedback.

• In traditionally taught classes,
  – where the students get their major feedback only after their first midterm exam,
  – the average gain is $g \approx 0.23$.

• For the classes with an immediate feedback, the average gain is twice larger: $g \approx 0.48$.

• In this talk, we provide a possible geometric explanation for this doubling of the learning rate.
5. Why Geometry

- Learning means changing the state of a student.
- At each moment of time, the state can be described by the scores $x_1, \ldots, x_n$ on different tests.
- Each such state can be naturally represented as a point $(x_1, \ldots, x_n)$ in the $n$-dimensional space.
- In the starting state $S$, the student does not know the material.
- The desired state $D$ describes the situation when a student has the desired knowledge.
- When a student learns, the student’s state of knowledge changes continuously.
- It forms a (continuous) trajectory $\gamma$ which starts at the starting state $S$ and ends up at the desired state $D$. 
6. First Simplifying Assumption: All Students Learn at the Same Rate

- Some students learn faster, others learn slower.
- The above empirical fact, however, is not about their \textit{individual} learning rates.
- It is about the \textit{average} rates of student learning, averaged over all kinds of students.
- From this viewpoint, it makes sense to assume that all the students have the same average learning rate.
- In geometric terms, this means that the learning time is proportional to the length of the corresponding curve $\gamma$.
- We thus need to show that learning trajectories corr. to immediate feedback are, on average, twice shorter.
7. Second Simplifying Assumption: the Shape of the Learning Trajectories

- At first, a student has misconceptions about physics or calculus, which lead him in a wrong direction.
- We can thus assume that at first, a student moves in a random direction.
- After the feedback, the student corrects his/her trajectory.
- In the case of immediate feedback, this correction comes right away, so the students goes in the right direction.
- In the traditional learning, with a midterm correction:
  - a student first follows a straight line of length \( d/2 \) which goes in a random direction,
  - and then takes a straight line to the midpoint \( M \).
- Then, a student goes from \( M \) to the destination \( D \).
8. 3rd Simplifying Assumption: 1-D State Space

- We can think of different numerical characteristics describing different aspects of student knowledge.
- In practice, to characterize the student’s knowledge, we use a single number – the overall grade for the course.
- It is therefore reasonable to assume that the state of a student is characterized by only one parameter \( x_1 \).
- In case of immediate feedback, the learning trajectory has length \( d \).
- To make a comparison, we must estimate the length of a trajectory corresponding to the traditional learning.
- This trajectory consists of two similar parts: connecting \( S \) and \( M \) and connecting \( M \) and \( D \).
- To estimate the total average length, we can thus estimate the average length from \( S \) to \( M \) and double it.
9. Analysis: Case of Traditional Learning

- A student initially goes either in the correct direction or in the opposite (wrong) direction.
- Randomly means that both directions occur with equal probability 1/2.
- If the student moves in the right direction, she gets exactly into the desired midpoint $M$.
- In this case, the length of the $S$-to-$M$ part of the trajectory is exactly $d/2$.
- If the student starts in the wrong direction, he ends up at a point at distance $d/2$ – on the wrong side of $S$.
- Getting back to $M$ then means first going back to $S$ and then going from $S$ to $M$.
- The overall length of this trajectory is thus $3d/2$. 
10. Resulting Geometric Explanation

- Here:
  - with probability 1/2, the length is $d/2$;
  - with probability 1/2, the length is $3d/2$.

- So, the average length of the $S$-to-$M$ part of the learning trajectory is equal to
  \[
  \frac{1}{2} \cdot \frac{d}{2} + \frac{1}{2} \cdot \frac{3d}{2} = d.
  \]

- The average length of the whole trajectory is double that, i.e., $2d$.

- This average length is twice larger than the length $d$ corresponding to immediate feedback.

- This explains why immediate feedback makes learning, on average, twice faster.
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