Reward for Good Performance Works Better Than Punishment for Mistakes: Economic Explanation

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1. Reward vs. Punishment: An Important Economic Problem

- One of the most important issues in economics is how to best stimulate people’s productivity.
- What is the best combination of reward and punishment that makes people perform better.
- This problem rises not only in economics, it appears everywhere.
- How do we stimulate students to study better?
- How do we stimulate our own kids to behave better?
2. Empirical Fact

• A lot of empirical studies were done on this topic.
• Some of these studies were made by Nobelist Daniel Kahneman – one of the fathers of behavioral economics.
• Most confirm that reward for good performance, in general, works better than punishment for mistakes.
• But why?
• Like many facts from behavioral economics, this fact does not have a convincing theoretical explanation.
• In this talk, we provide a theoretical explanation for this empirical phenomenon.
3. What People Want

- People spend some efforts $e$.
- Based on results of these efforts, they get a reward $r(e)$.
- In the first approximation, we can say that the overall gain is the reward minus the efforts: $r(e) - e$.
- A natural economic idea is that every person wants to maximize his/her gain, i.e., maximize $r(e) - e$; so:
  - to explain why rewards work better than punishments,
  - we need to analyze what are the reward functions $r(e)$ corr. to the two reward strategies.
- We will use simplified “first approximation” models, providing qualitative understanding of the situation.
4. What Reward Function Corresponds to Rewarding Good Performance

• What does rewarding good performance mean?

• On the one hand:
  
  – if the performance is not good, i.e., if the effort $e$ is smaller than the smallest needed effort $e_0$,
  
  – there is practically no reward: $r(e) = r_+$ for some $r_+ \approx 0$.

• On the other hand:
  
  – the more effort the person uses, the larger the reward;
  
  – so, every effort beyond $e_0$ is proportionally rewarded, i.e., $r(e) = r_+ + c_+ \cdot (e - e_0)$, for some $c_+$. 
5. Rewarding Good Performance (cont-d)

- The constant $c_+$ depends on the units used for measuring effort and reward:
  - one unit of effort corresponds to $c_+$ units of reward.
- These two formulas can be combined into a single formula

$$r(e) = r_+ + \max(0, c_+ \cdot (e - e_0)) = r_+ + c_+ \cdot \max(0, e - e_0).$$
6. Rewarding Good Performance (cont-d)

- This dependence has the following form:

\[ r(e) \]

![Graph showing the relationship between r(e) and e, with a step function.]
7. What Can We Say About This Function

- It is easy to see that our function is *convex*.
- This means that for all $e' < e''$ and for each $\alpha \in [0, 1]$, we have

$$r(\alpha \cdot e' + (1 - \alpha) \cdot e'') \leq \alpha \cdot r(e') + (1 - \alpha) \cdot r(e'').$$
8. What Reward Function Corresponds to Punishing for Mistakes

• What does punishing for mistakes means?

• On the one hand:
  – if the performance is good, i.e., if the effort $e \geq e_0$, then there is no punishment, i.e., the reward remains the same: $r(e) = r_-$ for some constant $r_-$;

• On the other hand:
  – the fewer effort the person uses, the most mistakes he/she makes,
  – so the larger the punishment and the smaller the resulting reward;
  – so, every effort below $e_0$ is proportionally penalized, i.e., $r(e) = r_- - c_- \cdot (e_0 - e)$, for some $c_-$. 

9. Punishing for Mistakes (cont-d)

- The constant $c_-$ depends on the units used for measuring effort and reward:
  - one unit of effort corresponds to $c_-$ units of reward.
- These two formulas can be combined into a single formula

$$r(e) = r_- - c_- \cdot \max(0, e_0 - e) = r_- + c_- \cdot \min(0, e - e_0).$$
10. Punishing for Mistakes (cont-d)

- This dependence has the following form:

\[ r(e) \]

\[ e \]
11. What Can We Say About This Function

- It is easy to see that this function is *concave*.
- This means that for all $E' < E''$ and for each $\alpha \in [0, 1]$, we have
  \[ r(\alpha \cdot e' + (1 - \alpha) \cdot e'') \geq \alpha \cdot r(e') + (1 - \alpha) \cdot r(e''). \]
- Now, we are ready to present the desired explanation.
12. Known Properties of Convex and Concave Functions: Reminder

• It is known that:
  – every linear function is both convex and concave;
  – the sum of two convex functions is convex, and
  – the sum of two concave functions is concave.

• In particular, the linear function \( f(e) = -e \) is both convex and concave, thus:
  – when the function \( r(e) \) is convex, the sum \( r(e) + (-e) = r(e) - e \) is also convex; and
  – when the function \( r(e) \) is concave, the sum \( r(e) + (-e) = r(e) - e \) is also concave.
13. Convex and Concave Functions (cont-d)

• It is also known that:
  – for a convex function, the maximum on an interval is always attained at one of the endpoints;
  – for a concave function, its maximum on an interval is always attained at some point inside the interval.
14. Resulting Explanation

• A person selects the effort $e_0$ for which the expression $r(e) - e$ attains its largest possible value.

• Of course, people’s abilities are not unbounded, there are certain limits within which we can apply the efforts.

• Thus, possible value of the effort $e$ are located within some interval $[e, \bar{e}]$.

• When we reward for good performance, the corresponding function $r(e)$ is convex.

• Thus the difference $r(e) - e$ is convex.

• Therefore, the selected value $e_0$ coincides either with $\underline{e}$ or with $\bar{e}$.

• We can dismiss the case $e_0 = \underline{e}$ when the reward is so small that it is not worth spending any effort.
15. Resulting Explanation (cont-d)

- So, we can conclude that $e_0 = \bar{e}$, i.e., the person selects the largest possible effort.
- This is exactly what we wanted to achieve.
- On the other hand, when we punish for mistakes, the corresponding function $r(e)$ is concave.
- Thus the difference $r(e) - e$ is concave.
- Therefore, the selected value $e_0$ is always located inside the interval $[\underline{e}, \bar{e}]$: $e_0 < \bar{e}$.
- Thus, the person will not select the largest possible effort – which is exactly what we wanted to avoid.
- This indeed explains why rewarding for good performance works better than punishment for mistakes.
16. Discussion

- What if we have both reward for good performance and punishment for mistakes, i.e.,

$$r(e) = \text{const} + c_+ \cdot \max(0, e - e_0) + c_- \cdot \min(0, e - e_0)?$$

- In this case, for $c_+ > c_-$, the function is still convex, i.e., we still get a very good performance.

- However, if $c_- > c_+$, the function becomes concave, and the performance suffers.

- Thus, to get good results, reward must be larger than punishment.
17. Discussion (cont-d)

- It is worth mentioning that:
  - the optimal rewarding function
    \[ r(e) = r_+ + c_+ \cdot \max(0, e - e_0), \]
  - in effect, coincides (modulo linear transformations of input and output)
  - with the efficient “rectified linear” activation function \( r(e) = \max(0, e) \) used in deep learning.
- So, not only people learn better when we use this function – computers learn better too!
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