Maximum Entropy in Support of Semantically Annotated Datasets

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1. Checking Whether Two Datasets Represent the Same Data: Formulation of the Problem

- *In the semantic web:* data are often encoded in Resource Description Framework (RDF).
- *In RDF:* every piece of information is represented as a triple consisting of a *subject*, a *predicate*, and an *object*.
- *Example:* a predicate *hasGravityReading*.
- *Problem:* in different datasets $D'$, $D''$ the same predicate *hasGravityReading* may not mean the same thing.
- *Existing solution:* use semantics.
- *Remaining problem:* concepts may still be slightly different.
- *Possible solution:* compare values $x'_1, \ldots, x'_n \in D'$ and $x''_1, \ldots, x''_n \in D''$ measured at the same locations.
2. Need to Take Uncertainty into Account

- **Problem** (reminder): check whether the predicate means the same in databases $D'$ and $D''$.

- **Solution** (reminder): compare values $x'_1, \ldots, x'_n \in D'$ and $x''_1, \ldots, x''_n \in D''$ measured at the same locations.

- **Ideal case** (of exact values): if $\Delta x_i \overset{\text{def}}{=} x'_i - x''_i = 0$ for all $i$, the predicate means the same in $D'$ and $D''$.

- **Problem**: due to measurement errors, the measurement result $x'_i$ differs from the actual (unknown) value $x_i$:

  $$\Delta x'_i \overset{\text{def}}{=} x'_i - x_i \neq 0.$$  

- **Hence**: $\Delta x_i = (x'_i - x_i) - (x''_i - x_i) = \Delta x'_i - \Delta x''_i \neq 0$.

- **Traditional assumption**: $\Delta x'_i$ are normally distributed, with 0 mean and known standard deviation $\sigma'_i$.

- **Conclusion**: $\sigma_i^2 = (\sigma'_i)^2 + (\sigma''_i)^2 + 2r_i \cdot \sigma'_i \cdot \sigma''_i$, where $r_i \in [-1, 1]$ is the correlation between $\Delta x'_i$ and $\Delta x''_i$. 
3. First Idea: Assume Independence

- **Reminder:** $\sigma_i^2 = (\sigma'_i)^2 + (\sigma''_i)^2 + 2r_i \cdot \sigma'_i \cdot \sigma''_i$, with the unknown correlation $r_i$.

- **Usual approach:** assume independence: $r_i = 0$ and $(\sigma_i)^2 = (\sigma'_i)^2 + (\sigma''_i)^2$.

- **Informal justification:**
  - all we know: $r_i \in [-1, 1]$;
  - information is invariant w.r.t. $T: r_i \rightarrow -r_i$;
  - conclusion: the selected $r_i$ must be invariant: $Tr_i = r_i$, so $-r_i = r_i$, and $r_i = 0$.

- **Formal justification:** the Maximum Entropy approach.

- **$\chi^2$ criterion:** if $\sum_{i=1}^{n} \frac{(\Delta x_i)^2}{(\sigma'_i)^2 + (\sigma''_i)^2} \leq \chi^2_{n, \alpha}$, then the two datasets $D'$ and $D''$ describe the same quantity.

- **Reminder:** $\sigma_i^2 = (\sigma'_i)^2 + (\sigma''_i)^2 + 2r_i \cdot \sigma'_i \cdot \sigma''_i$, with the unknown correlation $r_i$.

- **Previous approach:** assume independence ($r_i = 0$).

- **Problem:** measurement errors may be correlated.

- **Property:** if data fit for some values $\sigma_i$, then it fits for larger values $\sigma_i$ as well.

- **Resulting solution:** check the largest possible values $\sigma_i$.

- **Fact:** $\sigma_i$ is largest when $r_i = 1$; then $\sigma_i^2 = (\sigma'_i + \sigma''_i)^2$.

- **New $\chi^2$ criterion:** if $\sum_{i=1}^{n} \frac{(\Delta x_i)^2}{(\sigma'_i + \sigma''_i)^2} \leq \chi^2_{n,\alpha}$, then the two datasets $D'$ and $D''$ describe the same quantity.

- **Comment:** if this inequality is not satisfied, then the datasets describe somewhat different quantities.
5. Conclusion

• **Question:** are the semantically equivalent quantities in two databases $D'$ and $D''$ actually the same?

• **Input:**
  
  – semantically annotated measurement results $x'_1, \ldots, x'_n \in D'$ and $x''_1, \ldots, x''_n \in D''$;
  
  – information about the measurement uncertainty: st.dev. $\sigma'_i$ and $\sigma''_i$.

• **Case of independent measurement errors:** $D'$ and $D''$ represent the same data $\iff \sum_{i=1}^{n} \frac{(\Delta x'_i)^2}{(\sigma'_i)^2 + (\sigma''_i)^2} \leq \chi^2_{n,\alpha}$.

• **Alternative situation:** measurement errors may be correlated.

• **Recommendation:** $D'$ and $D''$ represent the same data $\iff$ a weaker inequality holds: $\sum_{i=1}^{n} \left( \frac{\Delta x'_i}{\sigma'_i + \sigma''_i} \right)^2 \leq \chi^2_{n,\alpha}$. 
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