How to Make a Decision
Under Set Uncertainty

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- To describe people’s preferences, it is convenient to assign a number called *utility* to each alternative, so that:
  - to better alternatives
  - we assign larger numbers.

- Then, we can select the alternative with the largest possible utility value.

- In practice, we rarely know the exact consequences of each action.

- So instead of the exact utility, we get a set \( S \) of possible values of utility.

- Without losing generality, we can assume that the set \( S \) is closed, i.e., that it contains all its limit points.
2. Decisions Under Set Uncertainty (cont-d)

- It turns out that it is reasonable to select an alternative that maximizes its **Hurwicz value**

\[ H(S) \overset{\text{def}}{=} \alpha \cdot \sup S + (1 - \alpha) \cdot \inf S. \]

- Here the parameter \( \alpha \in [0, 1] \) describes the person’s degree of optimism:
  - complete optimists correspond to \( \alpha = 1 \),
  - complete pessimists to \( \alpha = 0 \), and
  - everyone else to intermediate values of \( \alpha \).

- When two sets has the same Hurwicz value, then:
  - for most people (namely, for risk-averse ones),
  - it is reasonable to select the alternative for which the interval \([\inf S, \sup S]\) is the narrowest.
3. Formulation of the Problem

- But what if we have two sets:
  - with the same Hurwicz value and the same width,
  - i.e., for which the infimum and the supremum are the same?
- Traditional decision theory treats them as equally good.
- However, intuitively, a set \{0, 0.9, 1\} is better than \{0, 0.1, 1\}.
- Indeed, the first and third options are the same, but the second option is better for the second set.
- How can we describe this intuitive idea?
- In this talk, we show how to do it for finite sets.
4. Our Idea: Case of Finite Sets

- Let us pick an interval \([a, \bar{a}]\) and consider all sets for which \(\inf S = a\) and \(\sup S = \bar{a}\).
- How do we compare two such sets \(S_1\) and \(S_2\)?
- Let us first consider the case when both \(S_i\) are different from \(\{a, \bar{a}\}\).
- Then each set \(S_i\) is obtained by adding some elements to this 2-element set, i.e., we have
  \[
  S_i = \{a, \bar{a}\} \cup A_i, \text{ where } A_i \subseteq (a, \bar{a}).
  \]
- It is therefore reasonable to select the set \(S_i\) for which the subset \(A_i\) is preferable – i.e.,
  \[
  - \text{ either has larger Hurwicz value,}
  - \text{ or has same Hurwicz value and is narrower,}
  - \text{ or is better according to our new criterion.}
  \]
5. Our Idea: Case of Finite Sets (cont-d)

- If one of the sets – e.g., $S_1$ – is equal to $\{a, \overline{a}\}$, then:
  - if $H(A_2) < H(\{a, \overline{a}\})$, this means that in $S_2$, we add a worse alternative to $\{a, \overline{a}\}$;
  - thus, $S_1$ is better.

- If $H(A_2) > H(\{a, \overline{a}\})$, then similarly $S_2$ is better.

- If $H(A_2) = H(\{a, \overline{a}\})$, then $A_2$ is better than $\{a, \overline{a}\}$ – since it is narrower.

- So also $S_2$ is better.

- This way, for every two finite sets $S_i$, we:
  - either decide which is better
  - or reduce the problem to comparing sets with fewer elements.

- Since we consider finite sets, this procedure will eventually stop and we will decide which set is better.
6. General Case

- In general, when sets $S_i$ can be infinite.

- We can consider, for each $n$, $\left(\frac{1}{n}\right)$-approximations $S_{1,n}$ and $S_{2,n}$ to these sets.

- These sets are formed by elements $\frac{k}{n}$, where $k = \text{round}(n \cdot u)$ for some $u \in S_i$.

- If $S_{1,n}$ is better than $S_{2,n}$ for all sufficiently large $n$, then we say that $S_1$ is better than $S_2$.

- Similarly, we can explain when $S_2$ is better than $S_1$. 
7. References
