Empirical Power Law for Company Losses:
A Probability-Based Explanation

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1. **Formulation of the Problem**

- Companies compete in the market.
- Both a given company and its competitors constantly develop new products.
- One of these products becomes a winner.
- The efforts of all other companies do not lead to success and thus, qualify as losses:
  - the more money and efforts the company invests in the development of the new products,
  - the higher the probability that this company will succeed.
- Vice versa, the fewer money is invested, the higher the probability of failure.
2. Formulation of the Problem (cont-d)

- Analysis of different companies shows that, on average:
  - the probability of failure \( p \) is approximately inversely proportional to
  - the overall investment \( I \) in development of new products.

- To be more precise, \( p \approx \frac{c}{I + I_0} \) for some constants \( c \) and \( I_0 \).

- The problem is that there is no convincing explanation for the above formula.

- In this talk, we provide such an explanation.
3. Comment

- Similar dependencies can be found in many application areas.

- Historically the first such dependence was Zipf’s law – first formulated in linguistics.

- This law states that if the text is large enough, then:
  - when we order words in the decreasing order of frequency,
  - the frequency $f_n$ of $n$-th word is approximately equal to $f_n \approx \frac{c}{n + n_0}$ for some constants $c$ and $n_0$. 
4. Comment (cont-d)

- Similar formulas work well:
  - when we sort cities in the decreasing order of their population,
  - when we sort companies in the decreasing order of their sizes,
  - when we sort papers by number of citations,
  - when we sort earthquakes by magnitude, etc.
5. Our Explanation

- Let $k$ denote the number of new products developed by a given company.
- Let $a$ be the average investment needed to develop a new product.
- Then, the overall company’s investment is equal to
  \[ I = a \cdot k. \]
- So, in terms of the investment $I$, the value $k$ has the form $k = I/a$.
- Let $C$ denote the average number of new products proposed by the competition.
- Then, the overall number of competing products is
  \[ k + C. \]
6. **Our Explanation (cont-d)**

- It is reasonable to assume that all these products are equally reasonable.
- Thus, each of these products has the same probability of becoming a commercial success.
- This probability is equal to $1/(k + C)$.
- The probability that the given company loses:
  - can thus be estimated as the probability that one of $C$ competitors’ products will succeed,
  - and is, therefore, equal to $p = \frac{C}{k + C}$.
- Substituting $k = \frac{I}{a}$ into this formula, we get
  $$p = \frac{C}{\frac{I}{a} + C}.$$
7. Our Explanation (cont-d)

• *Reminder:* \( p = \frac{C}{\frac{1}{a} + C} \).

• Multiplying both the numerator and the denominator by \( a \), we conclude that \( p = \frac{a \cdot C}{I + a \cdot C} \).

• So, we indeed get the desired expression \( p \approx \frac{c}{I + I_0} \), with \( c = I_0 = a \cdot C \).
8. References