How to Reconcile Physical Theories with the Idea of Free Will: From Analysis of a Simple Model to Interval and Fuzzy Approaches

Julio C. Urenda[1] and Olga Kosheleva[2]
Departments of [1]Mathematical Sciences and [2]Teacher Education
University of Texas, El Paso, TX 79968, USA
emails jcurenda@miners.utep.edu, olgak@utep.edu

1. Introduction

- *Free will: a natural idea.* If we walk to a corner, then we can turn right or cross the street.
- *Commonsense belief:* it is not possible to predict beforehand what exactly a person will do.
- *In classical physics:*
  - once we know the positions and velocities of all the particles,
  - we can uniquely predict the exact future locations and velocities of all the particles.
- *Problem:* can we reconcile physics with free will?
- *Clarification:* with 10^{23} particles, predictions are not practically possible.
- From the commonsense viewpoint, even a theoretical prediction probability is very disturbing.

2. Is quantum physics an answer?

- At first glance, it may look as if this problem disappears in quantum physics.
- Due to Heisenberg's principle, we cannot exactly predict both the location and the velocity.
- Schroedinger's equations describe how the state ("wave function") $\psi(x, t)$ changes with time t.
- These equations are deterministic
  - once we know the original state $\psi(x, t_0)$,
  - we can uniquely determine the future state.
- So, we can uniquely predict the probabilities.
- In particular, we we can predict (at least theoretically) the probability that a person turns right.
- This also contradicts to common sense.

3. The problem of free will in physics has been actively studied in philosophy of physics

- *Mainstream approach:*
  - keep the physics as is;
  - commonsense intuition is faulty.
- *Argument:* quantum mechanics showed that commonsense intuitions are only approximately correct.
- *Alternative approach* (Penrose et al.): we need to modify our physical theories.
- *Problem:* no well-developed physical theory is fully consistent with our free will intuition.

4. Interval and fuzzy approaches: towards reconciliation between physics and free will

- *Traditional approach:* differential equations.
- *Idea:* the rate of change is uniquely determined by the state: $\dfrac{d\vec{v}_i}{dt} = \vec{F}_i(\vec{r}_1, \ldots, \vec{r}_n, \vec{v}_1, \ldots, \vec{v}_n)$.
- *Conclusion:* no free will.
- *Corollary:* to get free will, we must allow several possible values of rate of change.
- *Natural idea:* interval of possible values:

$$\frac{dv_{ia}}{dt} \in [\underline{F}_{ia}(\vec{r}_1, \ldots, \vec{r}_n, \vec{v}_1, \ldots), \overline{F}_{ia}(\vec{r}_1, \ldots, \vec{r}_n, \vec{v}_1, \ldots)].$$

- *Alternative idea:* several intervals corresponding to different degrees of certainty.
- Such nested intervals can be viewed as α-cuts of a fuzzy set, so we get fuzzy differential inclusions.

5. What we plan to describe

- Our objective is
  - to *reasonably* modify the equations of physics
  - so that it will be possible to make the motion no longer uniquely predictable.
- *In plain terms:* a physically explicit free will would mean that
  - by simply exercising our will,
  - we can actually change the motion of the physical particles.
- We would like to check if this is indeed possible within a meaningful physical theory.

6. Symmetry

- We need a theory which is consistent with free will.
- We want this theory to be physically meaningful.
- In modern physics, one of the most important notions is the notion of *symmetry*.
- The behavior of the physical particles must not change if we simply
  - shift them to a different spatial location,
  - or rotate the whole configuration,
  - or start the experiment at a later time moment.
- Thus, a meaningful physical theory must be invariant w.r.t natural symmetries.

7. Symmetries and conservation laws

- It is known that in physical equations, invariance with respect to symmetries lead to conservation laws:
  - invariance w.r.t. shifts in time means that energy $E = \sum_{i=1}^{n} \frac{1}{2} \cdot m_i \cdot (\vec{v}_i)^2$ must be preserved;
  - invariance w.r.t spatial shifts means that the (linear) momentum $\vec{p} = \sum_{i=1}^{n} m_i \cdot \vec{v}_i$ must be preserved;
  - invariance w.r.t. rotations means that the angular momentum $\vec{M} = \sum_{i=1}^{n} m_i \cdot (\vec{v}_i \times \vec{r}_i)$ must be preserved.
- Thus, we require that these three quantities are preserved in our physical theory.

8. Case of a single particle

- *Situation:* let us start our analysis with the case of a single particle.
- *Fact:* for this particle, the momentum $\vec{p} = m_1 \cdot \vec{v}_1$ is preserved.
- *Conclusion:* the velocity $\vec{v}_1$ is also preserved.
- *Conclusion:* no matter how much we exercise our will, this particle will not be diverted from its inertial path.
- So, for a single particle, no "true free-will" theory is possible:

9. Case of two particles

- Select the $t = t_0$ center of mass as the coordinates origin: $\dfrac{m_1 \cdot \vec{r}_1 + m_2 \cdot \vec{r}_2}{m_1 + m_2} = \vec{0}$, hence $\vec{r}_2 = -\dfrac{m_1}{m_2} \cdot \vec{r}_1$.
- Take a system that (originally) moves with the center: $m_1 \cdot \vec{v}_1 + m_2 \cdot \vec{v}_2 = \vec{0}$.
- Since the momentum $\vec{p} = m_1 \cdot \vec{v}_1 + m_2 \cdot \vec{v}_2$ is preserved, we have $m_1 \cdot \vec{a}_1 + m_2 \cdot \vec{a}_2 = 0$ hence $\vec{a}_2 = -\dfrac{m_1}{m_2} \cdot \vec{a}_1$.
- Since the angular momentum is preserved, we get $m_1 \cdot (\vec{a}_1 \times \vec{r}_1) + m_2 \cdot (\vec{a}_2 \times \vec{r}_2) = 0$, hence $\vec{a}_1 \times \vec{r}_1 = \vec{0}$.
- Thus, $\vec{a}_1$ is collinear with $\vec{r}_1$.
- Since energy $\sum \dfrac{\vec{v}_i^2}{2}$ is preserved, we get $\vec{a}_1 \cdot \vec{v}_1 = 0$ hence $\vec{r}_1 \cdot \vec{v}_1 = 0$ – which is in general not true.
- *Conclusion:* no free will for 2-particle systems.

10. Case of 3 particles: analysis

- For 3 particles, invariance leads to linear equations:

$$m_1 \cdot \vec{a}_1 + m_2 \cdot \vec{a}_2 + m_3 \cdot \vec{a}_3 = 0;$$

$$m_1 \cdot (\vec{a}_1 \times \vec{r}_1) + m_2 \cdot (\vec{a}_2 \times \vec{r}_2) + m_3 \cdot (\vec{a}_3 \times \vec{r}_3) = 0;$$

$$m_1 \cdot (\vec{a}_1 \cdot \vec{v}_1) + m_2 \cdot (\vec{a}_2 \cdot \vec{v}_2) + m_3 \cdot (\vec{a}_3 \cdot \vec{v}_3) = 0.$$

- We need to find three 3-D vectors $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$, i.e., $3 \cdot 3 = 9$ scalar unknowns.
- The first two equations are vector equations, each of which has 3 scalar components.
- So overall, we have 7 scalar equations to determine 9 (scalar) unknowns.
- Clearly, a linear system of 7 equations with 9 unknowns has many solutions.

11. General case: analysis

- For p particles, invariance leads to linear equations:

$$m_1 \cdot \vec{a}_1 + \ldots + m_p \cdot \vec{a}_p = 0;$$

$$m_1 \cdot (\vec{a}_1 \times \vec{r}_1) + \ldots + m_p \cdot (\vec{a}_p \times \vec{r}_p) = 0;$$

$$m_1 \cdot (\vec{a}_1 \cdot \vec{v}_1) + \ldots + m_p \cdot (\vec{a}_p \cdot \vec{v}_p) = 0.$$

- We need $3 \cdot p$ scalar parameters to determine p accelerations $\vec{a}_1, \ldots, \vec{a}_p$.
- We have the same number of 7 equations to satisfy.
- Since $3 \cdot p \geq 3 \cdot 3 > 7$, the corresponding linear system of equations always has a non-zero solution $\vec{a}_i$.
- *Conclusion:* for ≥ 3 particles, a "true free-will" physical theory is, in principle, possible.
- *Comment:* for 2 particles, we need to determine 6 scalar unknowns from $7 > 6$ equations.

12. General conclusion

- Physical theories (gravity, electrodynamics, etc.) are based on pairwise interactions between particles.
- *Example:* Newton's gravitation theory,

$$\vec{F}_i = \sum_{j \neq i} \frac{G \cdot m_i \cdot m_j \cdot (\vec{r}_j - \vec{r}_i)}{|\vec{r}_j - \vec{r}_i|^3}.$$

- In such theories, interaction between ≥ 3 bodies reduces to pairwise interaction.
- *Example:* Earth, Sun, and Moon resulting in tides.
- In a free-will theory, we must have triple interactions:

$$\vec{F}_i = \sum_{j \neq i} \vec{F}_{ij}(\vec{r}_i, \vec{r}_j, \vec{v}_i, \vec{v}_j) +$$

$$\sum_{j \neq i} \sum_{k \neq i} \vec{F}_{ijk}(\vec{r}_i, \vec{r}_j, \vec{r}_k, \vec{v}_i, \vec{v}_j, \vec{v}_k).$$

13. Discussion

- *Reminder:* for a true free-will theory, we need at least triple interactions.
- Similar cases when triple interactions bring complexity:
  - in decision making, combining 2 opinions is easy, but combining 3 leads to Arrow's paradox;
  - in celestial mechanics, 2-body problem is explicitly solved while a 3-body problem is complex.
- Possible use in fuzzy logic:
  - we normally use binary logical operations, fuzzy analogues of "or", "and", etc.;
  - more complex logical operations (e.g., ternary one) are usually reduced to the binary ones;
  - non-reducible ternary operations may lead to a more adequate representation of expert uncertainty.

14. Acknowledgments

This work was supported in part:

- by NSF grants HRD-0734825, EAR-0225670, and EIA-0080940,
- by Texas Department of Transportation contract No. 0-5453,
- by the Japan Advanced Institute of Science and Technology (JAIST) International Joint Research Grant 2006-08, and
- by the Max Planck Institut für Mathematik.

The authors are thankful

- to Professor John Symons from Philosophy Department and
- to the anonymous referees

for valuable suggestions.