A Path Following Method for Large-Scale and Dense $\ell_1$-Underdetermined Problems

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Abstract
In this work, we consider an homotopic principle for solving large-scale and dense $\ell_1$-underdetermined problems. The idea consists of obtaining the solution of the problem by solving a sequence of linear equality constrained multiquadratic problems that depends of a perturbed parameter that converges to zero. The procedure generates a “central” path that converges to a point on the solution set of the $\ell_1$-underdetermined problem. This allows to mimic the path-following methodology for primal-dual interior-point methods. To obtain inexact directions associated to the KKT conditions a fixed-point conjugate gradient method is implemented. To prevent the algorithm from becoming quite expensive, a measure of closeness to the “central” path is provided. The perturbed parameter is implemented in the same fashion as it is done in interior-point methods. To this end, we characterize the complementarity variables associated to the primal variables of the problem. We present a numerical result to recover sparse signals for some large scale problems, and compare our results with some state-of-the-art algorithms. Finally, we implement our algorithm successfully in some seismic reflection problems.