# Recent Results on the Asymptotics to a Class of Difference Equations of Monotone Type


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**Abstract**

We investigate the asymptotic behavior of bounded solutions to the following class of second order nonhomogeneous difference equations

$$\begin{cases} u_{n+1} - (1+\theta_n)u_n + \theta_n u_{n-1} \in c_n A u_n + f_n \quad n \geq 1 \\ u_0 = a \in H, \qquad \sup_{n\geq 0} |u_n| < +\infty \end{cases}$$

where $A$ is a maximal monotone operator in a real Hilbert space H, $\{c_n\}$ and $\{\theta_n\}$ are positive real sequences and $\{f_n\}$ is a sequence in $H$. With suitable conditions on $A$ and the sequences $\{c_n\}$, $\{\theta_n\}$ and $\{f_n\}$, we show the weak or strong convergence of $\{u_n\}$ or its weighted average to an element of $A^{-1}(0)$, which is also the asymptotic center of the sequence $\{u_n\}$, implying therefore in particular that the existence of a solution $\{u_n\}$ implies that $A^{-1}(0) \neq \emptyset$. Our results extend and give simpler proofs to previous results by several authors whose proofs use the assumption $A^{-1}(0) \neq \emptyset$. We also present some applications of our results to optimization.



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