

Why Feynman Path Integration?

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Need for quantization. Since the early 1900s, we know that to describe physics properly, we need to take into account quantum effects. Thus, for every non-quantum physical theory describing a certain phenomenon, be it mechanics or electrodynamics or gravitation theory, we must come up with an appropriate quantum theory.

Traditional quantization methods consist of replacing the scalar physical quantities with the corresponding operators. The problem with the above approach is that due to non-commutativity of the quantum operators, two mathematically equivalent formulations of the classical theory can lead to different (non-equivalent) quantum theories.

Feynman's approach based on the notion of the least action principle. Starting with Newton's mechanics, the laws of physics have been traditionally described in terms of differential equations, equations that explicitly describe how the rate of change of each quantity depends on the current values of this and other quantities. However, for *fundamental* physical phenomena, not all differential equations make sense: for example, we need equations that lead to conservation of fundamental physical quantities like energy, momentum, angular momentum, etc. It turns out that all known fundamental physical equations can be described in terms of an appropriate minimization principle – and, in general, equations following from a minimization principle lead to conservation laws.

For each fundamental physical theory, we can assign, to each trajectory $\gamma(t)$, we can assign a value $S(\gamma)$ such that among all possible trajectories, the actual one is the one for which the value $S(\gamma)$ is the smallest possible. This value $S(\gamma)$ is called *action*, and the principle that action is minimized along the actual trajectory is called the *minimal action principle*.

In Feynman's approach, the probability to get from the state $\underline{\gamma}$ to the state $\bar{\gamma}$ is proportional to $|\psi(\underline{\gamma} \rightarrow \bar{\gamma})|^2$, where $\psi = \sum_{\gamma: \underline{\gamma} \rightarrow \bar{\gamma}} \exp\left(i \cdot \frac{S(\gamma)}{\hbar}\right)$, the sum is taken over all trajectories γ going from $\underline{\gamma}$ to $\bar{\gamma}$, and \hbar is Planck's constant.

Feynman path integration is not just a foundational idea, it is actually an efficient computing tool: we can expand the corresponding expression and represent the resulting probability as a sum of an infinite series each term of which can be described by an appropriate graph called *Feynman diagram*.

Feynman's path integration: remaining foundational question. From the pragmatic viewpoint, Feynman path integral is a great success. However, from the foundational viewpoint, we still face an important question: why the above formula?

What we do in this talk. In this talk, we provide a natural explanation for Feynman's path integration formula.