Interval Newton Methods

Omeiza Olumoye* Martine Ceberio*†

In an attempt to find solutions to optimization problems, we are concerned about obtaining the absolute or global extrema amongst the possible extrema. A traditional numerical approach to solving optimization problems is the Newton method. The Newton method can be used in solving optimization problems by looking for lull or peak points of the objective function. The Newton method is quite simple to implement and converges fast (in fact \(q\)-quadratically) to one solution. This implies that the Newton method is a locally convergent method since it converges to a solution when the initial guess is “close” to that solution and otherwise it could diverge and not find any solution.

Since the Newton method can be used in solving optimization problems but it only converges to one solution, we are not guaranteed that the solution found will be the global solution. An ideal algorithm should be able to find the global solution irrespective of the initial guess - we don’t have to know a priori whether the initial guess is “close” enough to any solution. This begs the question of our needing an enhanced method that could find all possible solutions within a given interval. One such method is the Interval Newton Method, which is a globally convergent algorithm, converging to solutions from any arbitrary initial interval guess. In this case, we are assured to obtain a global solution to an optimization problem.

In this work, we will be presenting comparative numerical results of the traditional Newton method, as well as two variants of the Interval Newton Method - Newton method with intervals, and Krawczyk method, on various optimization problems.

*Computational Science Program, UTEP
†Computer Science, UTEP