Modal operators on rings of continuous functions

G. Bezhanishvili, L. Carai, and P. J. Morandi

Department of Mathematical Sciences
New Mexico State University

Abstract

The ring of real-valued continuous functions over a topological space has naturally a structure of $\ell$-algebra. $\ell$-algebras are hybrids between lattices and $\mathbb{R}$-algebras. Gelfand-Naimark-Stone duality provides a dual equivalence between the category of compact Hausdorff spaces and the category $\text{bal}$ of bounded archimedean $\ell$-algebras. Our goal is to generalize this duality to compact Hausdorff spaces endowed with continuous relations. Continuous relations correspond dually to modal operators on rings of real-valued continuous functions. More precisely, we provide a dual equivalence between the category $\text{KHF}$ of compact Hausdorff frames and the category $\text{mbal}$ of modal bounded archimedean $\ell$-algebras. In this talk, after giving the necessary definitions, we will show how to associate a modal operator to a relation and vice versa in order to obtain the duality between the categories $\text{KHF}$ and $\text{mbal}$. We will discuss the relation between this new duality and the well-known Thomason and Esakia-Goldblatt dualities from modal logic. The talk will be concluded by presenting some correspondence results.