Hybrid Schemes for Parameter Estimation Problems

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Global Optimization

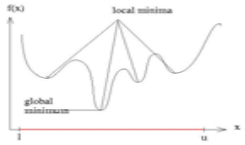
Problem: Find an optimal global solution for large-scale nonlinear parameter estimation problems, e.g., in Army Research Laboratory applications, which are difficult to solve due to large dense ill-conditioned operators and multiple non-optimal minima solutions among others.

Solution: Provide high-performance optimization procedures that combine the capabilities of stochastic and deterministic methods in searching for a global solution, and sensitivity analysis.

Global Optimization Problem

Given $f \in C^2$, $f : \Re^n \to \Re$ and $\Omega = \{x \in \Re^n : l \le x \le u\}$,
find $x^* \in \Omega$ such that $f(x^*) \le f(x)$ for all $x \in \Omega$.

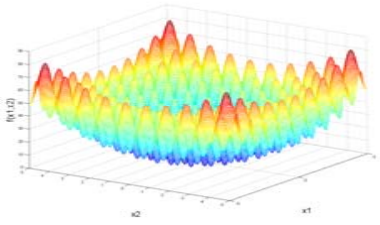
$$\underset{x \in \Omega}{\text{minimize}} \quad f(x)$$



Objective: Find a global minimizer of *f (x)* in the feasible set Ω

Global Solution

Example of global search: a function with a large number of local minima and maxima



Optimization Schemes

Current Approach = Multi-Start + Local Method + Step-length Control

Hybrid Approach = Multi-Start + Global Method + Surrogate Model + Local Method

where

Multi-Start: Set of random initial points

Local Method: Newton-Type, Levenberg-Marquardt, and Newton-Krylov Interior-Point (NKIP)

Global Method: Simulated Annealing, Simultaneous Perturbation Stochastic Algorithm, Global Levenberg-Marquardt

Hybrid Scheme

Global Method: SPSA

We consider a global derivative-free optimization method that uses only objective function values. The global method is to be efficient in high-dimensional problems, and perform a stochastic search to find target regions where the function value is low, and to allow us to conjecture in which region the global solution may lie.

Surrogate Model

Step 1: Find the surrogate model $f_s(x_k)$ using an interpolation method with the data:

$$(x_k,\ f(x_k)),\quad k = 1,\ldots,\quad p$$

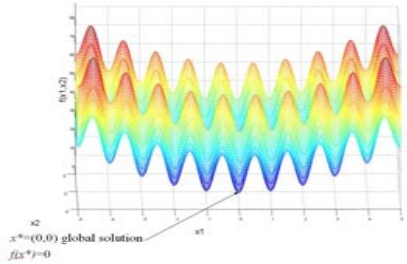
provided by the global method
Step 2: Obtain first/second order information with less computational cost without evaluating the real model
Step 3: Apply a local method to speed up the convergence to the global solution

Local Method: NKIP

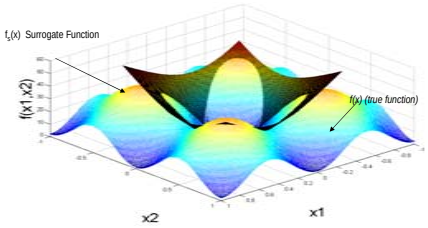
NKIP is a globalized and derivative dependent optimization method based on the global strategy introduced by [3]. This method calculates the directions using the conjugate gradient algorithm, and a linesearch is implemented to guarantee a sufficient decrease of the objective function. Most importantly, it allows the inclusion of equality constraints.

Idea

2-D Projection



Surrogate Model



Current and Future Work

Current work:
- Conduct preliminary numerical results of the hybrid optimization schemes on medium-scale parameter estimation problems
- Create an interface that allows the hybrid scheme to be model independent

Future work:
- Parallelize the entire hybrid optimization scheme
- Conduct sensitivity analysis

References

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