Automata, Computability and Formal Languages — Fall 2020
CS 3350 — L. Longpré

Pumping Lemma example

Pumping Lemma
If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0, xy^i z \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

A non regular language
Let $C = \{w \mid w$ has an equal number of 0s and 1s$\}$.

Theorem.
C is not regular.

Proof using the pumping lemma.
Leading to a contradiction, assume C is regular. Then, the pumping lemma applies to C. Let p be the pumping length that applies to C from the pumping lemma. Let $s = 0^p 1^p$. Since s has length at least p and is in C, the pumping lemma applies to s, so s can be divided into three strings $s = xyz$ where the conditions 1, 2 and 3 of the pumping lemma hold for x, y and z. Since conditions 2 and 3 hold, we know x and y contain only 0s and y contains at least one zero. Then $xyyz$ is not in C since it contains more 0s than 1s, contradictiing condition 1 of the pumping lemma for $i = 2$.

Proof using closure properties.
Leading to a contradiction, assume C is regular. Since regular languages are closed under intersection, then $C \cap 0^*1^*$ is also regular. But $C \cap 0^*1^* = \{0^n 1^n | n \geq 0\}$, which we already proved is not regular.