Automata, Computability and Formal Languages — Spring 2020
CS 3350 — L. Longpré

Assignment 4, solution

1. The string reversal operator s^R reverses the string.

   For example, $(0100011)^R = 1100010$, and $(\text{bravo})^R = \text{ovarb}$.

   The language reversal operator is defined as follows: $L^R = \{w^R \mid w \in L\}$.

   Show that regular languages are closed under reversal.

   We show how to transform a regular expression for L into (recursively) a regular expression for L^R.

   $\emptyset^R = \emptyset$

   $\varepsilon^R = \varepsilon$

   $a^R = a$ for $a \in \Sigma$

   $(r_1 \cup r_2)^R = r_1^R \cup r_2^R$

   $(r_1 \cdot r_2)^R = r_2^R \cdot r_1^R$

   $(r_1^*)^R = (r_1^R)^*$

   Another way is to show how to transform an NFA N for L into an NFA for L^R. Reverse all the arrows in N, create a new state S, make S the new start state, make ε transitions from the new start state to all final states. Then make F contain only the old start state.

2. Use the closure of regular language under reversal to prove that the following language is not regular:

   $L = \{1^n 0^n | n \geq 0\}$.

   Leading to a contradiction, assume L is regular. Since regular languages are closed under reversal, then L^R is also regular. But $L^R = \{0^n 1^n | n \geq 0\}$, which we already proved is not regular.