

Sample midterm 2 exam questions

1. Give an example of each of the following. In all questions, the alphabet is $\{a, b\}$. If no example exists, answer “impossible”. No justification needed.
 - (a) A pushdown automaton for the language $a^+ \varepsilon$.
 - (b) A context-free grammar with 2 variables, two terminals, two rules and one start variable.
 - (c) An ambiguous context-free grammar for the language $a^+ \varepsilon$.
 - (d) A parse tree for the string $abbba$ generated by the CFG:
 $S \rightarrow aSa \mid SS \mid b$.
 - (e) An ambiguous string generated by the CFG: $S \rightarrow aSa \mid SS \mid b$.
 - (f) A pushdown automaton that accepts everything except ε
 - (g) Two different languages that are not regular
2.
 - (a) Build a context-free grammar for the language $\{a^n b^{2n} \mid n \geq 2\}$.
 - (b) Build a parse tree for the string $aaabbbbb$ according to your grammar.
3. The following context-free grammar has some ε production rules, but cannot generate ε . Use the algorithm we saw in class to build an equivalent context-free grammar without ε production rules.

$$\begin{aligned} S &\rightarrow aXYa \\ X &\rightarrow aX \mid Xa \mid b \mid Y \mid Z \\ Y &\rightarrow YbY \mid bZ \mid X \mid \varepsilon \\ Z &\rightarrow bb \mid XXa \end{aligned}$$

4. Complete the statement of the pumping lemma for regular languages.

If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p , then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

- (a) for each $i \geq 0$, ...
- (b) $|y| \dots$
- (c) $|xy| \dots$

5. Questions like assignments 5, 6 and 7.