# UNIFYING DEPENDENT CLUSTERING AND DISPARATE CLUSTERING FOR NON-HOMOGENEOUS DATA

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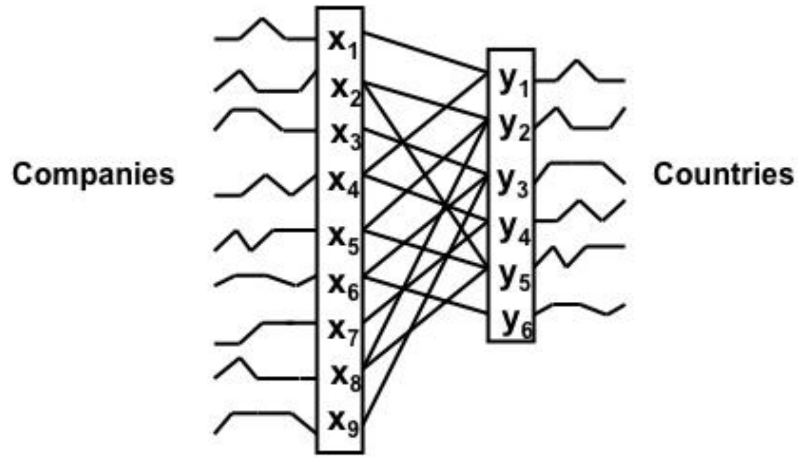
# Problem Setting

| Companies | Avg. salary of Employees | Stock values | Profit margins |
|---|---|---|---|
| $x_1$ | 1.0 K | 25.11 | 11% |
| $x_2$ | 1.1 K | 21.32 | 20% |
| $x_3$ | 1.2 K | 28.81 | 12% |
| $x_4$ | 1.2 K | 31.85 | 22% |
| $x_5$ | 1.1 K | 85.32 | 5% |
| $x_6$ | 1.2 K | 10.71 | 32% |
| $x_7$ | 0.9 K | 11.61 | 18% |
| $x_8$ | 1.1 K | 35.81 | 12% |
| $x_9$ | 1.2 K | 20.81 | 4% |

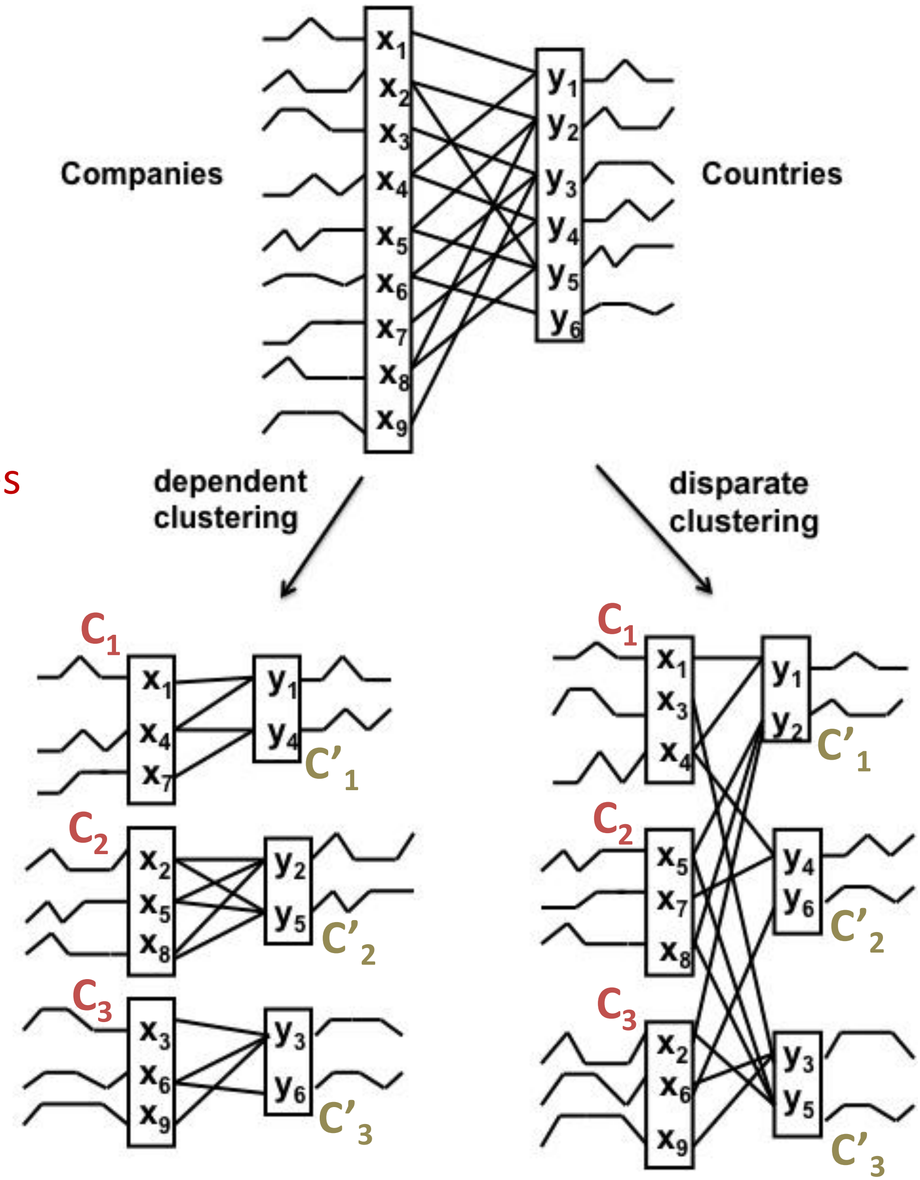
| Countries | GDP | GNP | Inflation |
|---|---|---|---|
| $y_1$ | \$11832 B | \$12970 B | -0.4% |
| $y_2$ | \$8219 B | \$8153 B | 2.0% |
| $y_3$ | \$6732 B | \$7812 B | -0.3% |
| $y_4$ | \$1761 B | \$2852 B | 1.8% |
| $y_5$ | \$5022 B | \$4391 B | 0.0% |
| $y_6$ | \$7224 B | \$8312 B | 1.1% |

# Objective



# Objective

Fortunes of individual companies are inter-twined with the fortunes of the countries.




Performances of companies may not necessarily be tied to the economies of the countries.

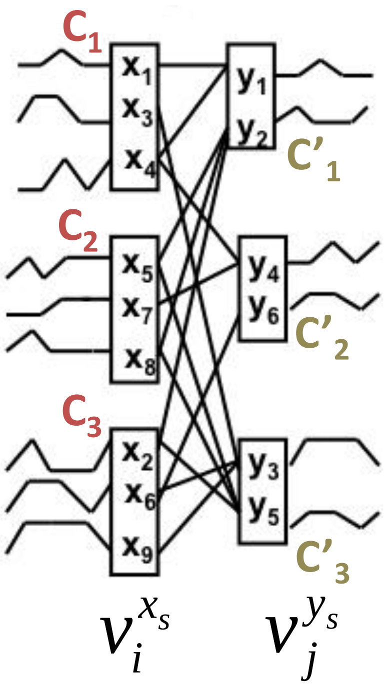
Companies × Countries (dependent clustering):

| | $C'_1$ | $C'_2$ | $C'_3$ |
|---|---|---|---|
| $C_1$ | 4 | 0 | 0 |
| $C_2$ | 0 | 6 | 0 |
| $C_3$ | 0 | 0 | 4 |

Companies × Countries (disparate clustering):

| | $C'_1$ | $C'_2$ | $C'_3$ |
|---|---|---|---|
| $C_1$ | 2 | 1 | 1 |
| $C_2$ | 2 | 1 | 2 |
| $C_3$ | 1 | 1 | 3 |

# Objective Function




| | C'1 | C'2 | C'3 |
|---|---|---|---|
| C1 | 2 | 1 | 1 |
| C2 | 2 | 1 | 2 |
| C3 | 1 | 1 | 3 |

**Obj = $\mathcal{F}$(Contingency table)**

**= $\mathcal{F}$(*n*(Clustering1, Clustering2, Relation))**

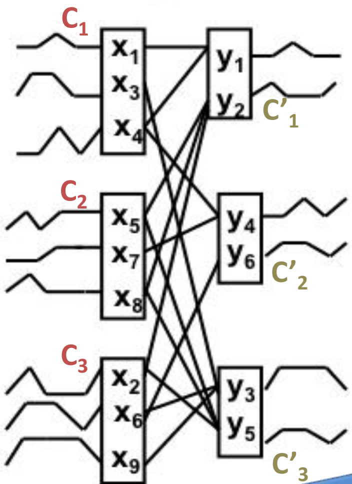
**= $\mathcal{F}$(*n*(*v*(Dataset1, Prototypes1),**

**\ \ \ \ *v*(Dataset2, Prototypes2),**

**\ \ \ \ Relation))**

- Optimize $\mathcal{F}$
  - Disparate clustering:
    - minimize: $\mathcal{F}$
  - Dependent clustering:
    - maximize: $\mathcal{F}$ or
    - minimize: -$\mathcal{F}$
- Quasi Newton Trust Region Algorithm

# Formulations




|  | C'1 | C'2 | C'3 |
|---|---|---|---|
| C1 | 2 | 1 | 1 |
| C2 | 2 | 1 | 2 |
| C3 | 1 | 1 | 3 |

*w*

|  | C'1 | C'2 | C'3 |
|---|---|---|---|
| C1 | 0.33 | 0.33 | 0.33 |
| C2 | 0.33 | 0.33 | 0.33 |
| C3 | 0.33 | 0.33 | 0.33 |

*U*

$\alpha$

|  | C'1 | C'2 | C'3 |
|---|---|---|---|
| C1 | 0.5 | 0.25 | 0.25 |
| C2 | 0.4 | 0.2 | 0.4 |
| C3 | 0.2 | 0.2 | 0.6 |

*Row distributions*

$\beta$

|  | C'1 | C'2 | C'3 |
|---|---|---|---|
| C1 | 0.4 | 0.33 | 0.17 |
| C2 | 0.4 | 0.33 | 0.33 |
| C3 | 0.2 | 0.33 | 0.5 |

*Col. distributions*

**Membership Probability**

$$v_i^{(\mathbf{x}_s)} = \frac{\exp(-\frac{\rho}{D}||\mathbf{x}_s - \mathbf{m}_{i,\mathcal{X}}||^2)}{\sum_{i'=1}^{k_x} \exp(-\frac{\rho}{D}||\mathbf{x}_s - \mathbf{m}_{i',\mathcal{X}}||^2)}$$

**Contingency Table Counts**

$$w_{ij} = \sum_{s=1}^{n_x} \sum_{t=1}^{n_y} B(s,t) v_i^{(\mathbf{x}_s)} v_j^{(\mathbf{y}_t)}$$

**Probability Distributions (row and column)**

$$p(\alpha_i = j) = p(C_{(y)} = j | C_{(x)} = i) = \frac{w_{ij}}{w_{i.}}$$

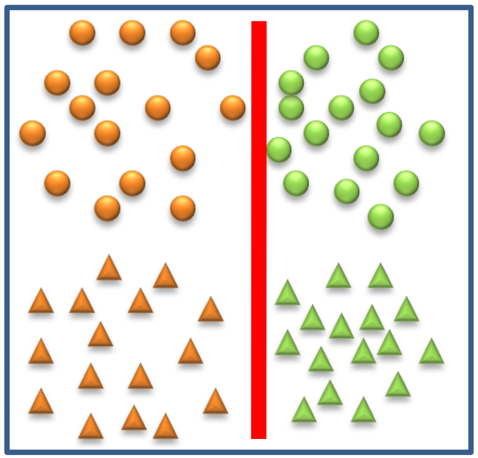
$$p(\beta_j = i) = p(C_{(x)} = i | C_{(y)} = j) = \frac{w_{ij}}{w_{.j}}$$

**Objective Function**

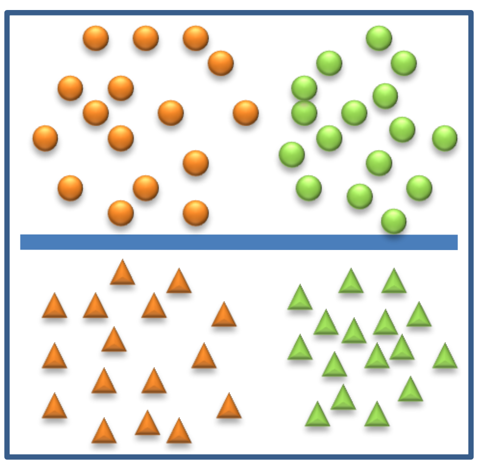
$$\mathcal{F} = \frac{1}{k_x} \sum_{i=1}^{k_x} D_{KL}\Big(\alpha_i || U(\frac{1}{k_y})\Big) + \frac{1}{k_y} \sum_{j=1}^{k_y} D_{KL}\Big(\beta_j || U(\frac{1}{k_x})\Big)$$

# Single Dataset Scenarios

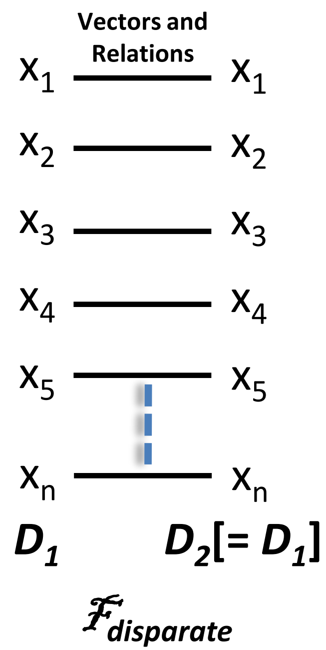
- **ALTERNATIVE CLUSTERING**



$D_1$

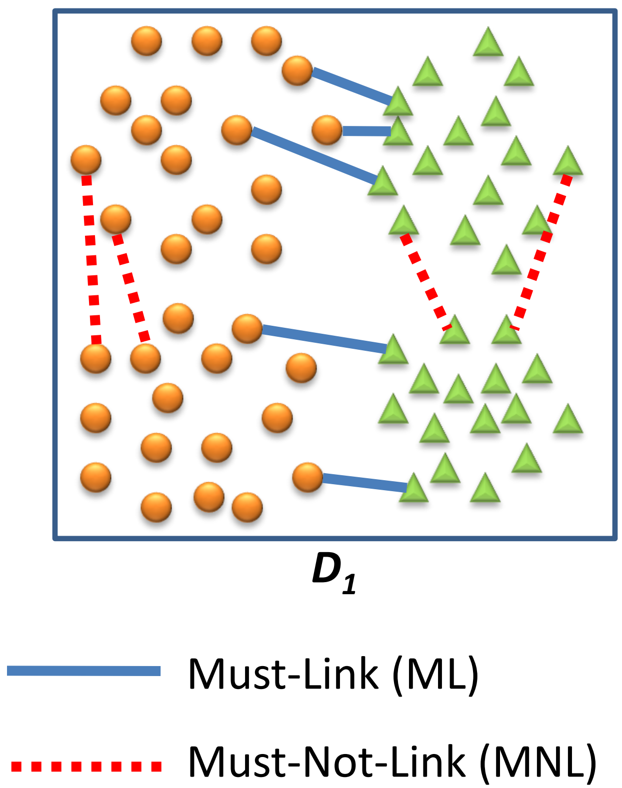


$D_2$

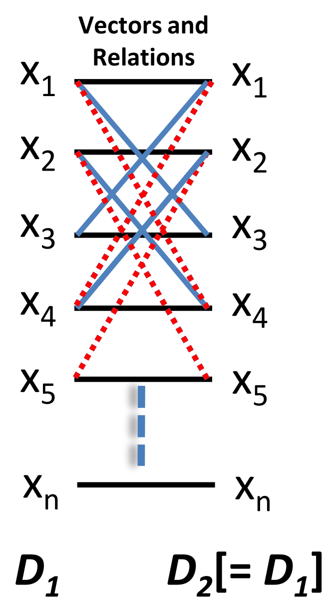


# Single Dataset Scenarios

- **CONSTRAINED CLUSTERING**
  - Instance-level constraints



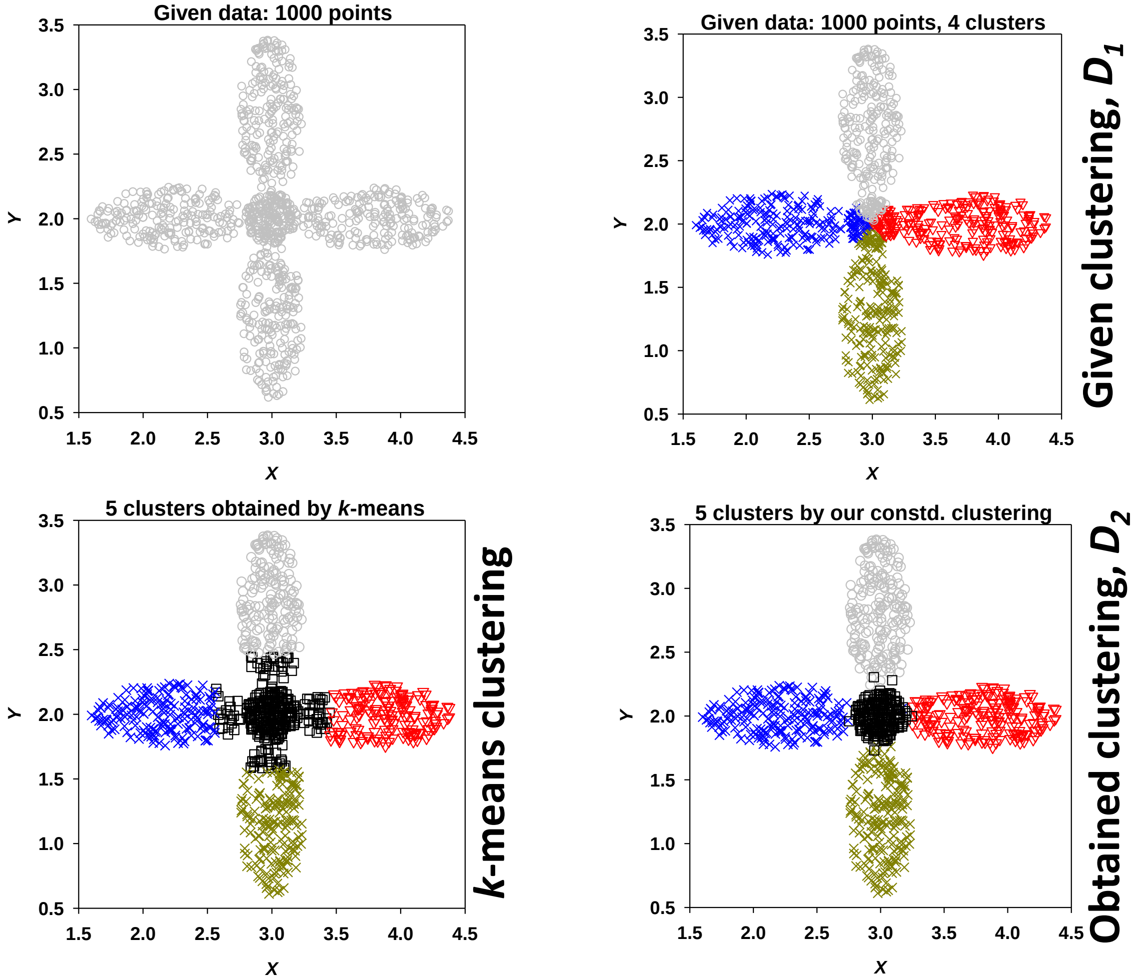

- **Clustering of $D_1$ is given.**
- **The desired constrained clustering is obtained in $D_2$.**




$$\mathcal{F} = \alpha\,\mathcal{F}_{dep} + (1-\alpha)\,\mathcal{F}_{disparate}$$

# Single Dataset Scenarios

- **CONSTRAINED CLUSTERING**
  - Cluster-level constraints

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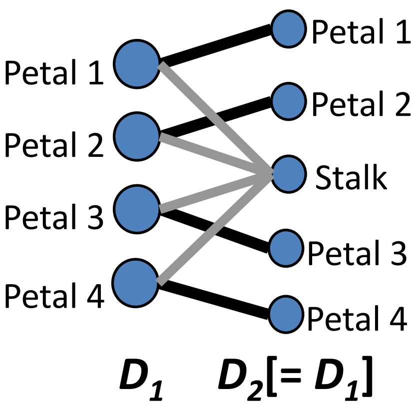


# Experimental Results

- **ALTERNATIVE CLUSTERING**
- Portrait Dataset

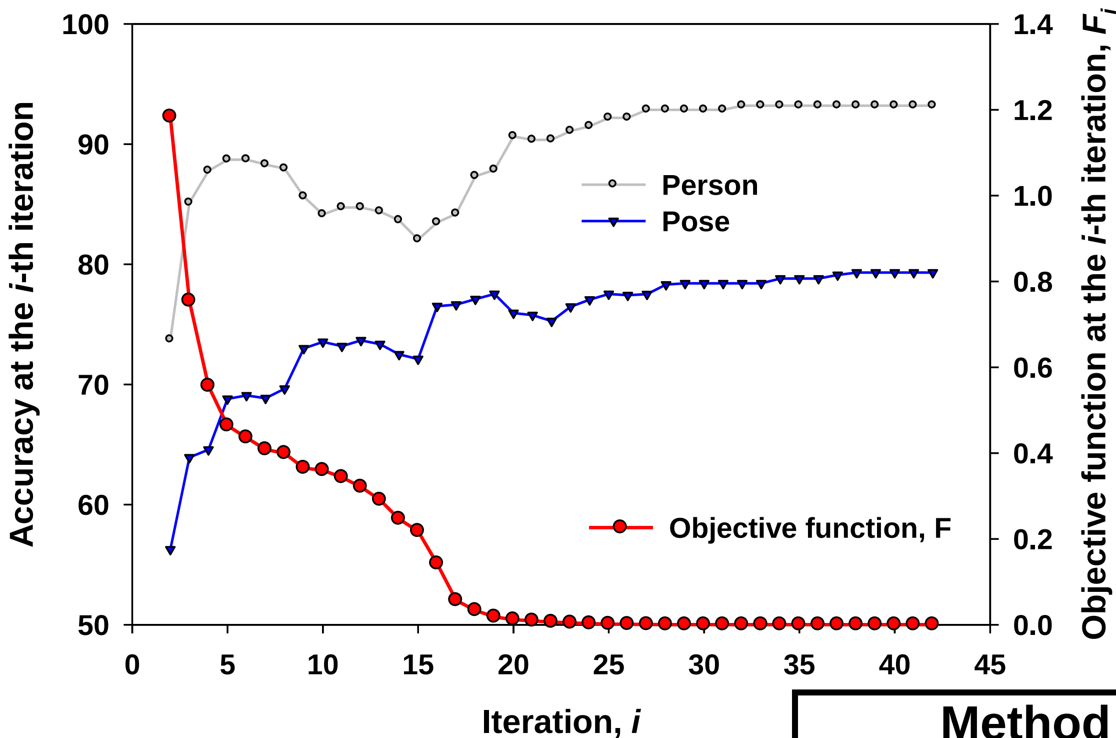
Prateek Jain et al. 2008

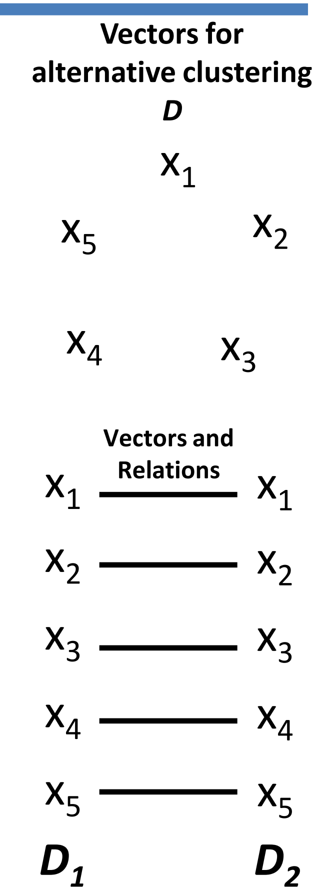
- **3** people each in **3** poses and **36** illuminations (i.e., **324** images.)
- **300** features

# Experimental Results

- **ALTERNATIVE CLUSTERING**

Portrait dataset, Iterations=42
$\text{Accuracy}_{person}$=93%, $\text{Accuracy}_{pose}$=79%
(Accuracy axis is at left and the axis for objective function is at right)

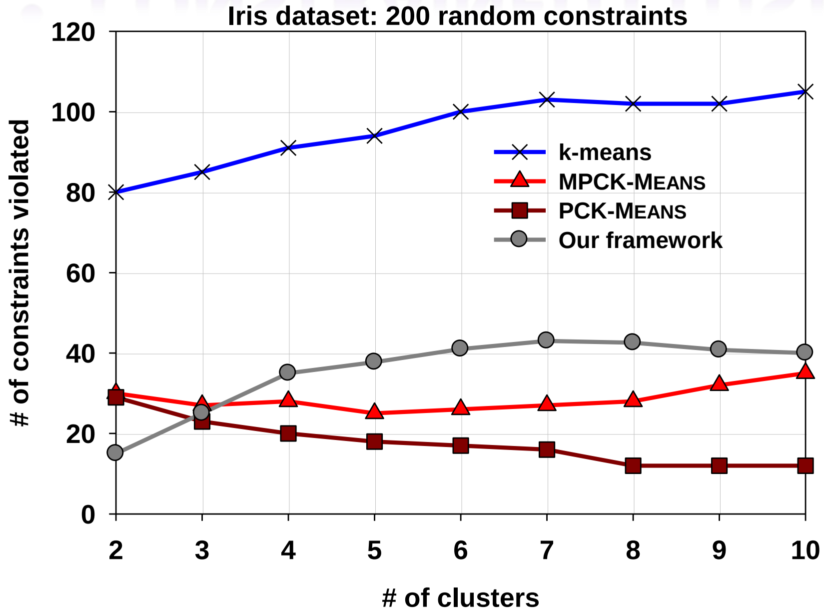


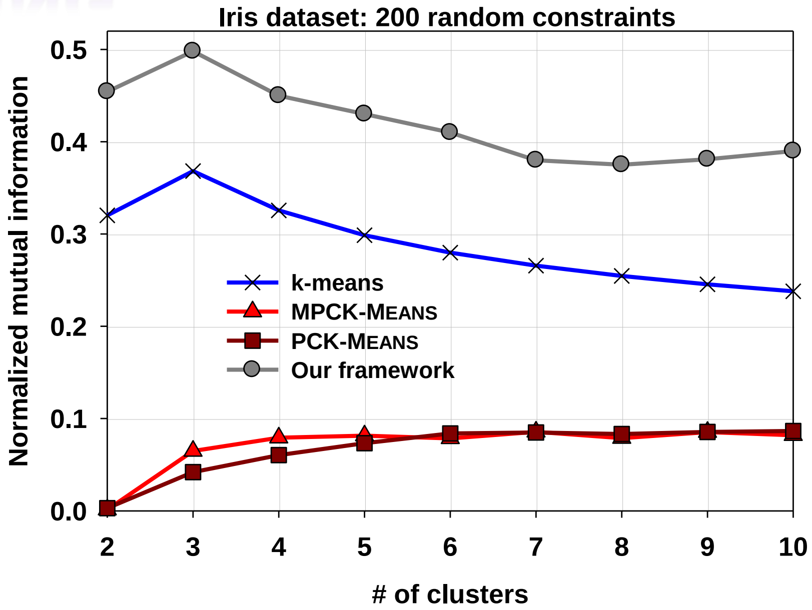



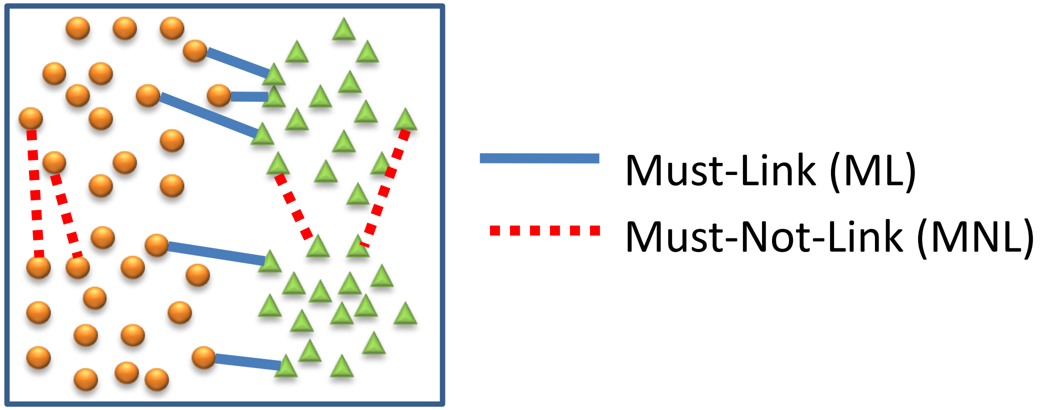

| Method | Person | Pose |
|---|---|---|
| *k*-means | 0.65 | 0.55 |
| Conv-EM | 0.69 | 0.72 |
| Dec-kmeans | 0.84 | 0.78 |
| Our framework | 0.93 | 0.79 |

# Experimental Results

- **CONSTRAINED CLUSTERING**



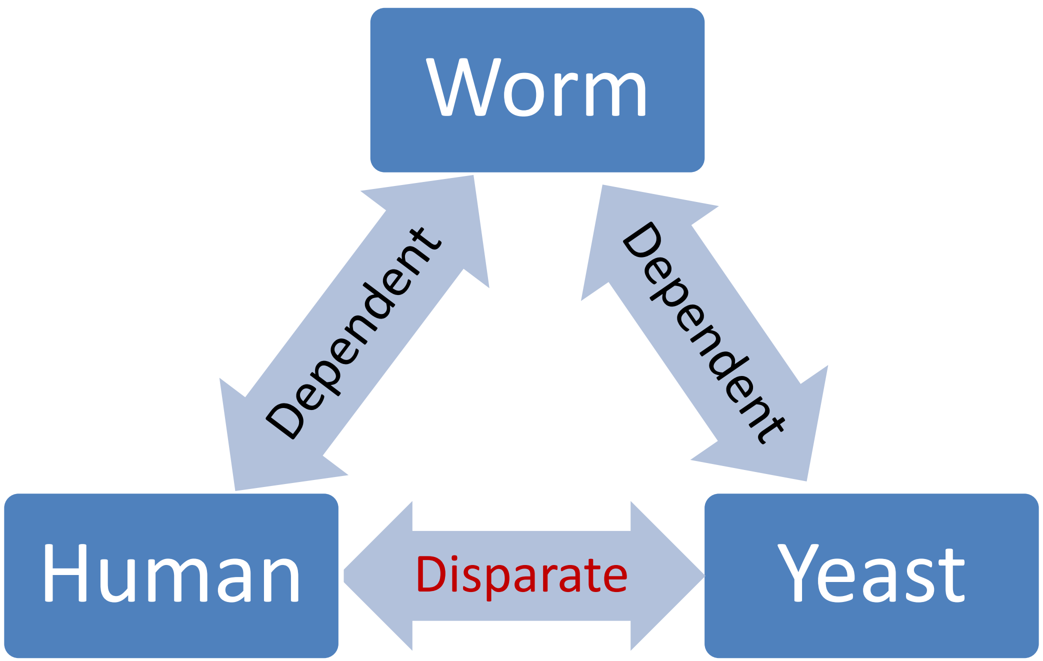

$D_1$

# Experimental Results

- **COMPARING GENE EXPRESSION PROGRAMS**

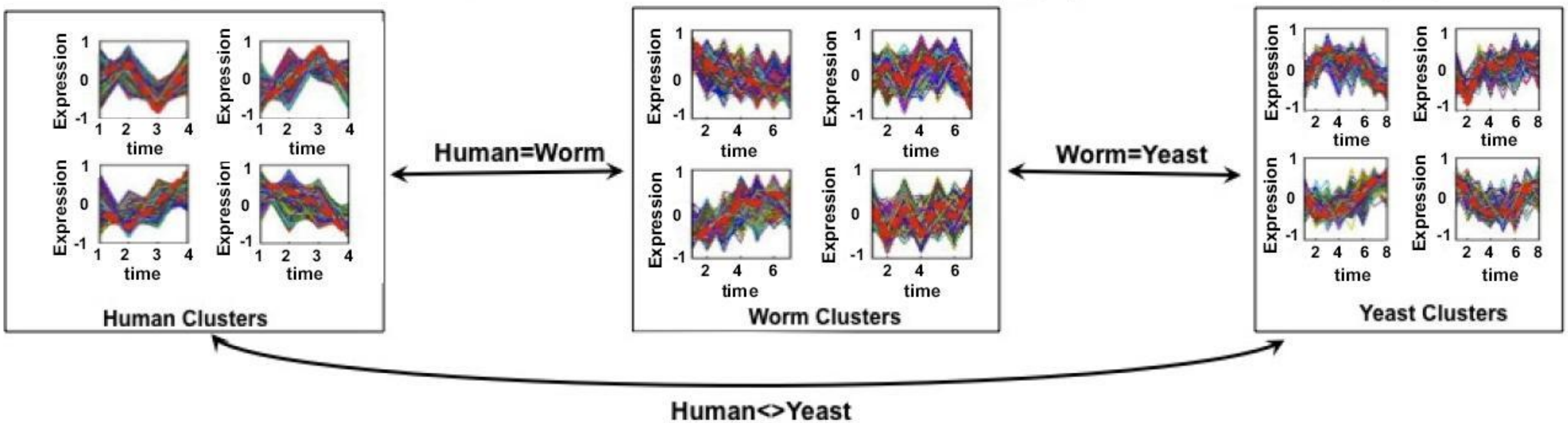
| Gene | Count |
|---|---|
| Human | 9,125 |
| Yeast | 3,664 |
| Worm | 5,987 |

| Pairs | Relationships |
|---|---|
| Human-Worm | 12,000 |
| Worm-Yeast | 8,002 |
| Human-Yeast | 9,012 |



# Experimental Results

- **COMPARING GENE EXPRESSION PROGRAMS**




| | | | |
|---|---|---|---|
| 468 | 1302 | 383 | 1006 |
| 126 | 1447 | 261 | 839 |
| 1198 | 1601 | 105 | 1046 |
| 1005 | 580 | 430 | 205 |

Human=Worm (original)

| | | | |
|---|---|---|---|
| 2308 | 522 | 73 | 264 |
| 922 | 1695 | 199 | 192 |
| 301 | 985 | 1979 | 291 |
| 48 | 596 | 453 | 1168 |

Human=Worm (final)

| | | | |
|---|---|---|---|
| 393 | 573 | 266 | 230 |
| 69 | 612 | 808 | 819 |
| 850 | 38 | 703 | 586 |
| 182 | 666 | 455 | 752 |

Worm=Yeast (orignal)

| | | | |
|---|---|---|---|
| 1061 | 120 | 160 | 307 |
| 401 | 1360 | 121 | 273 |
| 281 | 411 | 1197 | 423 |
| 158 | 311 | 391 | 1027 |

Worm=Yeast (final)

| | | | |
|---|---|---|---|
| 561 | 604 | 312 | 137 |
| 284 | 1217 | 175 | 813 |
| 926 | 41 | 578 | 757 |
| 1034 | 47 | 800 | 726 |

Human<>Yeast (orignal)

| | | | |
|---|---|---|---|
| 298 | 537 | 388 | 499 |
| 318 | 376 | 698 | 588 |
| 807 | 452 | 622 | 781 |
| 606 | 729 | 454 | 859 |

Human<>Yeast (final)

# Future Work & Conclusion

- Future directions
  - Capture more expressive relationships
    - Dependent and disparate clustering on same set of relationships
    - Different goal for different types of relationships (one-to-one, ML, MNL, etc.)
  - Clustering dependencies
- Conclusion
  - General, expressive framework for clustering non-homogenous datasets
  - The framework subsumes previously defined formulations
    - MDI (Kullback et al. ‘78), Disparate Clustering (Jain et al. ‘ 08), Clustering over Relation Graphs (Banerjee et al. ‘07), Multivariate Information Bottleneck (Friedman ‘01), etc.

# Thank you