Theorem. Let $L$ be a distributive lattice of cardinality $\leq \aleph_1$ such that if $a \in L$ then $\{b \mid b \leq a\}$ is countable. Then $L$ is an initial segment of the Turing Degrees.

The proof is similar to that in 79T-A-168 (misclassified). Also, refer to Lachlan's article in Zeitschrift vol. 14. A condition consists of a finite sublattice of $L$, $L_0$, a finite Boolean Algebra $B \bigcap L_0$, trees for each $a \in L_0$, and recursive increasing functions $f_{a,b}^\omega : \omega \to \omega$ for each $a \in L_0$, and atomic $\text{atom} b \leq a$ of $B$, such that (1) $\{f_{a,b}^\omega \mid b \leq a\}$ is a partition of $\omega$ for each $a \in L_0$, and $\omega \to \omega$ and $\omega \to \omega$

(2) if all $f$'s are defined then $f_{a,b}^\omega (m) < f_{a,b'}^\omega (n) = f_{a',b}^\omega (m) < f_{a',b'}^\omega (n)$

Lemma. Let $p = \langle L_0, B, T_a, f_{a,b}^\omega \rangle$ be a condition. Let $B' \bigcap B$ be a finite Boolean Algebra. Then, there exists a condition $q = \langle L_1, B', T_{a,b}^\omega, g_{a,b}^\omega \rangle$ such that $g_{a,b}^\omega (m) = f_{a,b}^\omega (n) b' \leq b$, and such that $\forall b' \exists b \exists g_{a,b}^\omega$ is infinite.

Lemma. Let $p = \langle L_0, B, T_a, f_{a,b}^\omega \rangle$ and $q = \langle L_1, B, S_a, g_{a,b}^\omega \rangle$ be conditions, with $L_1 \bigcap L_0$ and $p \geq q | L_0$ (as defined in 79-T-A-168). Then there exists a condition $r = \langle L_1, B, U_a^r, h_a^r \rangle$ such that $r \geq q$.

Definition. Let $M$ be a model of $BPA$ such that: $\langle a \rangle$ is possible. (Received September 17, 1979.) (Author introduced by Professor Leo Harrington.)

Theorem. There exists a provably hypersimple set $S$ and a model $M$ of $BPA$ such that:

(a) If $a \in N$ then $M \vdash a \in S$ iff $N \vdash a \in S$.

(b) If $M, S$ is decidable in near-linear time.

(c) In $M$, there exists a finite (bounded) set $F$ which is definable in the language of $M$ but undecidable in $M$.


2. Simmons, H. Existentially closed models of basic number theory. In: R. Gandy, M. Hyland (eds.), Logic Colloquium 76 North Holland (1977), 325-369. (Received September 18, 1979.) (Author introduced by V. Ja. Kreinovic.)
