OPTIMAL INTERVAL COMPUTATION TECHNIQUES: OPTIMIZATION OF NUMERICAL METHODS IN CASE OF UNCERTAINTY

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1. Formulation of the problem.

1.1. Traditional problem of interval computations research: designing a method.

Historically, in interval computations, the main emphasis was on developing *a* method for solving a given numerical problem: to compute a range $f(\mathbf{x}_1, \ldots, \mathbf{x}_n)$ of a given function f over given intervals $\mathbf{x}_1, \ldots, \mathbf{x}_n$; to compute the root of a given function; to solve a given system of equations, etc. In the recent past, the main user's problem was to find *an* algorithm for solving his problem.

1.2. New problem: choosing a method.

In course of time, many methods have been designed, and not often, the user faces a different problem: to *choose* between a variety of different interval techniques. The big problem here is: how should we choose?

1.3. This problem is difficult.

This problem is difficult first of all because it is difficult to formalize. A user would like the chosen method to be better "on average" on the problems that he will encounter, but usually, no reliable statistics on the frequencies of different problems is available. Moreover, these frequencies of different problems *vary* with time (in a hard-to-predict way), and, what makes the problem seemingly hopeless, these frequencies *depend* on what method we choose: if someone invents a good method of solving linear programming problems, then the frequency of such problems increase, because users *translate* their problems into linear programming in order to solve them.

1.4. This problem is important.

How are methods chosen now? The typical way is by using either intuition or *benchmark problems*. Both approaches are very subjective and therefore, error-prone.

It is therefore desirable to design a methodology of choosing the "best" (in some sense) numerical method in the situation of uncertainty (when no numerical optimality criterion $J(m)$ exists for choosing the best method m).

2. Our Idea: Group-Theoretic Approach to Optimization

In many cases, although we do not know the objective function $J(m)$, we do know that the problem possesses some natural symmetries (i.e., invariance with respect to rotations, changing measuring units, etc). In this case, we can assume that the (unknown) optimality criterion is invariant with respect to these transformations. This sounds like a very weak assumption at first glance, but in many cases this information about symmetries is sufficient to solve the optimization problem!

In this talk, we will present a survey of such situations:

- In [Gerasimov 1988], this idea was used for choosing the best interval-based model in *measurement theory*.
- In [Kreinovich 1994], this idea was used to choose the best interval model for *software testing*.
- In [Lea 1996], it was used for applications to interval-based *intelligent control*.
- In [Koltik 1986], this idea is used in the design of an optimal method for the problem that is similar to interval computations: the problem of estimating the *stochastic* error of indirect measurements.

Other applications will be described, including:

- Choosing the optimal *penalty function*.
- Choosing the optimal *set representation* of $n-$dimensional uncertainty (why ellipsoids?).
- Choosing the optimal $\varepsilon-$*inflation* (G. Mayer, 1996).

3. This Approach is Very Successful in Other Areas

As an additional argument in favor of this group-theoretic approach, we can say that this approach (described first in [Kozlenko 1989], [Kreinovich 1990], [Kreinovich 1990a]) is known to be very successful in different areas. For example:

- In *theoretical physics*, it can be used as a "major physical law" from which all fundamental physical equations can be derived [Finkelstein 1985], [Finkelstein 1986].
- In *intelligent control*, it leads to a control that is more stable and smooth [Smith 1995].
- In *expert systems*, this idea leads to the best choice of knowledge representation [Kozlenko 1988].
- In *genetic algorithms*, it leads to the best scaling.
- In *system theory*, it leads to the optimal description of the dynamics of the unknown system [Kozlenko 1988a].

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