GEOMETRIC APPROACH TO QUARK CONFINEMENT

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Abstract. Interactions that explain quark confinement correspond, in the first approximation, to interactions in 2-D space-time. In this paper, we show that this fact has a geometric explanation: Namely,

- on the most fundamental geometric level (of causality relations) quark confinement means that causality forms a *lattice* (i.e., an ordering in which every two events have the least upper bound and the greatest lower bound); and
- a space-time is a lattice if and only if its proper space is 1-D (i.e., if the space-time itself is 2-D).

1. INTRODUCTION

Quarks: brief success history and quark confinement problem. Quark theory, proposed by M. Gell-Mann in the 60s to explain the properties and interactions of hadrons and mesons, has been a great success. Practically all its predictions were experimentally confirmed, from the indirect ones (numerical values of particle characteristics) to the direct ones: that scattering amplitudes must behave as if a proton contains three particle-like parts (*partons*).

This experimental confirmation clearly indicated that quarks are not idealized mathematical constructions, helpful to describe real world, but they are *real* elementary particles bound together

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by some force. (It turned out that previously known interaction forces could not explain quarks interaction, so special interaction particles called *gluons* were invented to describe quark interaction.)

Traditionally in physics, such an *indirect* "proof of existence" of new particles (from the properties of their bounded states) was usually followed by a *direct* observation of these new particles in the free state: sometimes, by using an accelerator with energy sufficient to brake the bonds; sometimes, by observing these free particles in the cosmic rays, etc.

Unfortunately, all the efforts to *extract* the quarks from hadrons and mesons and/or directly observe them as *free* particles were unsuccessful. Quarks behaved as if they were *confined* in a hadron or in a meson.

1.2. Quantum Chromodynamics explains quark confinement on mathematical level; a more fundamental explanation is desired. Several theories have been proposed to explain this phenomenon of quark confinement. These efforts led to a theory called Quantum Chromodynamics (QCD) that is consistent with all known observations and that explains why free quarks are impossible (see, e.g., (Quark 1977), (Yndurain 1983), (Astbury 1995)). From the *mathematical* viewpoint, the problem of quark confinement is solved.

However, from the *physical* viewpoint, the situation is not completely satisfactory: Indeed, QCD is not a final theory because it does not describe *all* physical interactions and particles; hence, we must *generalize* it. From the purely mathematical viewpoint, every mathematical theory has a huge amount of different generalizations; the only way to restrict this variety is by applying physical intuition.

To use this intuition, we need, in addition to mathematical equations, to be able to formulate the main ideas behind them in simple physical terms.

Generalizations of physical theories are often based on *abandoning* some previous physical concepts: e.g., special relativity got rid of absolute time, quantum mechanics – of the precise values of the particle's coordinate and momentum. So, if we want our physical explanations to be as general as possible, we must formulate them in terms of the most *fundamental* physical concepts, i.e., the concepts that are most likely to remain physically meaningful in the generalized theory.

An ideal explanation should only use causality. One of the most fundamental physical notions is the notion of *causality*. Many notions can be described in terms of causality: e.g., from the causality relation of special relativity, we can uniquely determine the linear structure on the space-time Alexandrov 1950, Alexandrov 1953, Zeeman 1964, Naber 1992, Kreinovich 1994). The successful reformulation of different physical concepts in terms of causality relation lead many physicists to believe that causality is the "only physical variable" in the sense that everything else can be described in terms of it. Therefore, the ideal, most fundamental explanation of quark confinement must be in terms of causality only.

What we are planning to do. We start the paper with the description of the main idea of the existing physical explanation (in Section 2), and show that these explanations can be reformulated in terms of the most fundamental, geometric notion: the notion of causality (in Section 3). For this reformulation, we will need a mathematical theorem; for reader's convenience, the proof of this theorem is placed in a separate section (Section 4).

2. QUARK CONFINEMENT: THE MAIN PHYSICAL IDEA

Quark confinement is a non-relativistic phenomenon. Since quarks are confined within hadrons and mesons, theirs speeds are much smaller than the speed of light, and so, we can use non-relativistic physics to describe their interaction.

3-D non-relativistic fields (described by traditional field equations) cannot explain quark confinement. In non-relativistic physics, interaction can be described by fields with quanta of different rest mass m_0. The potential function $\varphi(\vec{r})$ of a field with quanta of rest mass m_0 is described by an equation

$$\triangle\varphi + m_0^2\varphi = 0.$$

In 3-D space, the fundamental solution of this equation is well known:

$$\varphi(\vec{r}) = \frac{\exp(-m_0 r)}{r}.$$

This potential function tends to 0 as $\vec{r} \to \infty$; therefore, if we have a particle bound by such interaction, then a finite amount of energy can allow this particle to get out of the potential well. In other words, standard 3-D interactions cannot explain quark confinement.

Main idea: to describe quark interaction, we need space of dimension $d \neq 3$. Since we cannot explain quark confinement by interactions in a 3-D space, it is natural to try to describe it by assuming that locally, the space dimension d is different from 3.

Only 1-D space can explain quark confinement. For $d > 3$, we also have $\varphi \to 0$; hence, to explain confinement, we must take $d < 3$, i.e., $d = 1$ or $d = 2$.

For $d = 2$, the fundamental solution for a massless interaction field ($m_0 = 0$) is $\varphi(\vec{r}) = \ln(r)$. The corresponding potential energy $E = Q\varphi(\vec{r}) = Q \cdot \ln(r)$ does tend to ∞ as $r \to \infty$, so, from the purely mathematical viewpoint, we do have confinement in the sense that for any given amount of energy E, the particle remained confined to a finite region; the radius r of this region can be determined from the equation $E = Q \cdot \ln(r)$. The resulting radius $r = \exp(E/Q)$ grows exponentially with E and therefore, does not describe the observed fact that the quarks are confined to a small region whose radius does not grow that drastically with the increase in energy.

So, the only remaining case if $d = 1$. In this case, the equations of massless interaction lead to $\varphi(\vec{r}) = r$. The resulting potential energy grows fast with r, and this growth explains why quarks cannot be separated.

In view of this explanation, the formula $\varphi(r) = r$ is used as a first approximation to the description of gluonic potential.

3. QUARK CONFINEMENT IN TERMS OF THE MOST FUNDAMENTAL NOTION: CAUSALITY

Confinement reformulated in terms of causality: main idea. Confinement means that although quarks that form, say, a meson, are independent particles, located at different spatial points, they cannot be separated in the sense that:

- their *joint* influence on any future event can be explained as if these quarks formed a *single* particle; and
- the influence of any part event on these two quarks can also be explained as if these quarks formed a single particle.

Formal description of this idea: causality must be a lattice ordering. In the above description, "influence" means any type of influence, i.e., influence means *causality* relation. So, to describe the above idea formally, we can use the standard denotation $a \preceq b$ as a shorthand for "event a can causally influence event b".

In terms of $\preceq$, this idea sounds as follows:

1. For every event e_1 (= point in space-time) on the first quark and for every event e_2 on the second quark, there exists a (fictitious) single event e^+ such that for any future event f, we have $e_1 \preceq f$ and $e_2 \preceq f$ (i.e., f is influenced by both events e_i) iff $e^+ \preceq f$.
2. Similarly, for every event e_1 on the first quark and for every event e_2 on the second quark, there exists an event e^- such that for any past event p, we have $p \preceq e_1$ and $p \preceq e_2$ (i.e., p influences both events e_i) iff $p \preceq e^-$.

Quarks are bound no matter where exactly they are located inside the meson (or a hadron). Therefore, the property described above must be true for every two events e_i in this area. In other words, locally, the causality relation must satisfy the following two properties:

1. For every two events e_1 and e_2, there exists an event e^+ such that for any other event f, $e_1 \preceq f$ and $e_2 \preceq f$ iff $e^+ \preceq f$.
2. For every two events e_1 and e_2, there exists an event e^- such that for any other event p, $p \preceq e_1$ and $p \preceq e_2$ iff $p \preceq e^-$.

In mathematical theory of orderings:

- the element e^+ that satisfies the first property called a *union* (or *least upper bound*) of the two elements e_1 and e_2, and denoted by $e_1 \vee e_2$;
- the element e^- is called an *intersection* (*meet*, or *greatest lower bound*) of the two elements e_1 and e_2 and denoted by $e_1 \wedge e_2$;
- an ordered set in which for every two elements, there exists a union and an intersection, is called a *lattice*.

So, we can conclude that *quark confinement means that the causality relation on a space-time forms a lattice.*

The natural question is: how is this conclusion related to the above explanation in terms of dimension d of (proper) space?

First (simple) result: when the space is Euclidean, causality is a lattice iff $d = 1$. If the space itself is Euclidean, i.e., if we take a $(d+1)-$dimensional Minkowski space-time with the usual causality relation

$$(t, x_1, \ldots, x_d) \preceq (t', x'_1, \ldots, x'_d) \leftrightarrow$$
$$t' - t \geq \sqrt{(x'_1 - x_1)^2 + \ldots + (x'_d - x_d)^2},$$

then, as is well known, this causality relation is a lattice only when $d = 1$.

For $d = 1$, this relation can be reformulated as

$$(u, v) \preceq (u', v') \leftrightarrow u \leq u' \,\&\, v \leq v',$$

where $u = t + x_1$ and $v = t - x_1$, and therefore, for every two events $e_1 = (u_1, v_1)$ and $e_2 = (u_2, v_2)$, we can define their union $e_1 \vee e_2 = (\max(u_1, u_2), \max(v_1, v_2))$ and their intersection $e_1 \wedge e_2 = (\min(u_1, u_2), \min(v_1, v_2))$.

This result is not fully realistic. At first glance, this simple result reformulates quark confinement in terms of the most fundamental relation, the causality:

- we started with the description of quark confinement in terms of causality, and
- we showed that this description is equivalent to space being (locally) 1-D: this is exactly what is needed to describe quark confinement in terms of non-relativistic field theory.

However, this result is not fully realistic: namely, we assumed that the space is Euclidean, but at the same time, since we assume that the space is assumed to be 1-D in the vicinity of a meson or a hadron and 3-D in other places, it is not exactly a typical flat Euclidean space.

To make this reformulation more convincing, it is therefore desirable to prove that this equivalence of lattice causality and 1-D character of space is true for curved spaces as well.

This is what we will do in the remaining part of the paper.

Motivations of the following model. Let us denote space by S, and the distance between spatial points s and s' by $d(s, s')$. In mathematical terms, we assume that S is a *metric space*, i.e.:

- that $d(s, s') = 0$ iff $s = s'$;
- that d is *symmetric*, i.e., $d(s, s') = d(s', s)$; and
- that d satisfies the triangle inequality $d(s, s'') \leq d(s, s') + d(s', s'')$.

Since for quarks, gravity is negligible, and gravity corresponds to curving *time* (or, to be more precise, space-*time*), we can safely assume that the time is not curved and can be therefore described by special relativity-like formulas.

In other words, time can be described by a temporal coordinate t, and we can assume that the speed of light c (maximal possible velocity that determines the causality relation) is constant.

Here, events are pairs (t, s), where t is a real number ($t \in R$) and $s \in S$. So, the space-time, i.e., the set of all possible events, is the set of all possible pairs (t, s), or, in mathematical terms, a Cartesian product $R \times S$ of the sets R and S.

An event (t, s) can causally influence an event (t', s') iff during the time $t' - t$ between these two events, some effect of the first event could have reached the second event. The largest distance that the effect can travel during the time $t' - t$ is $c(t' - t)$. Therefore, the above description of causality can be reformulated as follows:

$$(t, s) \preceq (t', s') \leftrightarrow c(t' - t) \geq d(s, s'). \tag{1}$$

Historical comment. If d is an Euclidean metric, this is a causality relation of special relativity. For a general metric space S, this (very natural) definition was first formulated by H. Busemann in (Busemann 1967).

The question is: *to describe metric spaces S for which the causality relation (1) on $R \times S$ is a lattice.* We will show that all such spaces are 1-D.

Since we consider very general spaces, we must specify what we mean by 1-D.

What is 1-D? Discussion. Among planar geometric objects (i.e., points, straight lines, planes, hyperplanes, etc), a 1-D set (i.e., a line) can be characterized by requiring that for every three points, one of them is between the two others. In terms of Euclidean metric, the fact that s' is "between" s and s'' can be described as follows: $d(s, s'') = d(s, s') + d(s', s'')$.

Strictly speaking, this was a definition of a "no more than 1-dimensional" set, which could possible be 1-D or 0-D (i.e., a point). To exclude a point (or a discrete set of points) and thus, to guarantee that we indeed have a 1-D space, we must also require that the set is *convex*, i.e., that with every two points, it contains the entire *interval*, i.e., the entire set of points between them.

For curved spaces, these definitions have to be slightly modified, because, e.g., a circle is clearly a 1-D space, but if we take 3 points at 120° from each other, then they do not satisfy the above equality. As a result, we arrive at the following definition:

Definitions of 1-D and the Main Result.

Definition 1. *We say that a point s' of a metric space S lies between s and s'' if $d(s, s'') = d(s, s') + d(s', s'')$.*

Definition 2. *We say that a metric space S is convex if for every $s, s'' \in S$ and for every real number d from the interval $[0, d(s, s'')]$, there exists a point s' between s and s'' for which $d(s, s') = d$.*

Definition 3. *We say that a metric space S is acyclic 1-dimensional if the following property is true: If s' is between s and s'', and s_0 is any other point from S, then:*

- *either s' is in between s and s_0,*
- *or s' is in between s'' and s_0.*

Comments. Clearly, the straight line and any of its intervals are acyclic 1-D spaces. These two examples do not exhauts all possible such spaces. Another example is a union of several intervals with a common point O, equipped with the following *internal* metric:

- the distance inside each of the intervals is standard, and
- if s and s' belong to different intervals, then $d(s, s')$ is defined as $d(s, O) + d(O, s')$.

It is easy to check that this space is:

- *convex* (because the segment $[s, s']$ between every two points is isomorphic to an interval) and
- *acyclic 1-dimensional* (because if s' is between s and s'', then s' belongs either to the same interval as s, or to the same interval as s'').

Theorem. *For every metric space S, the following two conditions are equivalent to each other:*

- *the space-time $(R \times S, \preceq)$ with with causality relation defined by formula (1) is a lattice;*
- *S is a convex acyclic 1-D space.*

4. PROOF

This proof will consists of two parts:

- First, we will prove that if $R \times S$ is a lattice, then X is convex and acyclic 1-D.
- Next, we will prove that if S is convex and acyclic 1-D, then $R \times S$ is a lattice.

First part of the proof. Let us assume that $R \times S$ is a lattice, and let us prove that S is convex and acyclic 1-D.

Proof of convexity. Let's first prove that S is convex.

Assume that $0 < d < d(s, s'')$. To prove that S is convex, we must construct a point s' for which $d(s, s') = d$ and $d(s', s'') = d''$, where we denoted $d'' = d(s, s'') - d$. To construct such a point s', let us consider the events $(d/c, s)$ and $(d''/c, s'')$ from the lattice $R \times S$. Since $R \times S$ is a lattice, there exists an intersection (meet) (t', s') of these two events. Let us show that s' is the desired point.

For convenience, let us denote $d' = ct'$, then $t' = d'/c$. The fact that the event $(d'/c, s')$ is an intersection means that the following two statements hold:

1. the event $(d'/c, s')$ *precedes* each of the events, i.e., $(d'/c, s') \preceq (d/c, s)$ and $(d'/c, s') \preceq (d''/c, s'')$; and
2. the event $(d'/c, s')$ is the *largest* of all events that precede both $(d/c, s)$ and $(d''/c, s'')$, i.e., if $(d_1/c, s_1) \preceq (d/c, s)$ and $(d_1/c, s_1) \preceq (d''/c, s'')$, then $(d_1/c, s_1) \preceq (d'/c, s')$.

In the following proof, we will use the first statement (we will mark this part of the proof by 1.), and then, twice, we will use the second of these statements; the corresponding parts of the proof will be marked 2.1 and 2.2.

1. The two inequalities from the *first statement*, according to (1), mean that

$$d - d' \geq d(s, s') \tag{1}$$

and

$$d'' - d' \geq d(s', s''). \tag{2}$$

2.1. Substituting $s_1 = s$ into the *second statement*, we conclude that if $d_1 \le d$ and $d_1 \le d'' - d(s, s'')$, then $d_1 \le d' - d(s, s')$. Hence, if $d_1 \le \min(d, d'' - d(s, s''))$, then $d \le d' - d(s, s')$. In particular, for $d_1 = \min(d, d'' - d(s, s''))$, we conclude that

$$\min(d, d'' - d(s, s'')) \le d' - d(s, s'). \qquad (3)$$

By definition of d'', we have $d'' = d(s, s'') - d$, hence, $d'' \le d(s, s')$, $d'' - d(s, s'') \le 0 \le d$, so $\min(d, d'' - d(s, s'')) = d'' - d(s, s'')$, and the inequality (3) takes the form $d'' - d(s, s'') \le d' - d(s, s')$, which is equivalent to

$$d'' - d' \le d(s, s'') - d(s, s'). \qquad (4)$$

From the triangle inequality, we can conclude that $d(s, s'') - d(s, s') \le d(s', s'')$. Using (4), (2), and this inequality, we can now conclude that

$$d'' - d' \le d(s, s'') - d(s, s') \le d(s', s'') \le d'' - d'.$$

In this chain of inequalities, the first and the last term coincide; therefore, all inequalities in thsi chain are actually equalities:

$$d'' - d' = d(s, s'') - d(s, s') = d(s', s'').$$

The second of these equalities means

$$d(s, s') + d(s', s'') = d(s, s''), \qquad (5)$$

i.e., that s' is in between s and s''. To complete the proof, it is sufficient to show that $d(s, s') = d$.

Indeed, we have already shown that

$$d'' - d' = d(s', s''). \qquad (6)$$

2.2. By considering $s_1 = s''$ in the *second statement*, we can similarly prove that

$$d - d' = d(s', s). \qquad (7)$$

Substituting (6) and (7) into (5), we conclude that $d'' + d - 2d' = d(s, s'')$. Substituing the definition of d'' (i.e., $d'' = d(s, s'') - d$) into this equation, we conclude that $d(s, s'') - 2d' = d(s, s'')$, i.e., that $d' = 0$. Hence, from (7), we conclude that $d(s, s') = d$. Indeed, s' is the desired point.

Proof of 1-dimensionality. Let us now show that S is acyclic and 1-D. Let s' be between s and s'', and let s_0 be any other point from S.

The fact that s' is in between s and s'' means that $d(s, s'') = d(s, s') + d(s', s'')$. Similarly to the previous part of the proof, let us denote $d = d(s, s')$ and $d'' = d(s', s'')$. Then, clearly, $(0, s') \preceq (d/c, s)$ and $(0, s') \preceq (d''/c, s'')$.

According to the previous section of the proof, the intersection of the two events $(d/c, s)$ and $(d''/c, s'')$ has the form $(0, \tilde{s}')$ for some $\tilde{s}' \in S$. Since this intersection is the largest of all the events that precede both $(d/c, s)$ and $(d''/c, s'')$, it is larger than $(0, s')$. Therefore, $(0, s) \preceq (0, \tilde{s}')$, which means that $0 \geq d(s', \tilde{s}')$. So, $d(s', \tilde{s}') = 0$, and $\tilde{s}' = s'$. Hence, the event $(0, s')$ is the intersection of (d, s) and (d'', s'').

Let us now apply the second property of intersection to $s_1 = s_0$. For $s_1 = s_0$, this property means that if $d_1 \leq d - d(s, s_0)$, and $d_1 \leq d'' - d(s_0, s'')$, then $d_1 \leq -d(s_0, s')$. In other words, if $d_1 \leq \min(d - d(s, s_0), d'' - d(s_0, s''))$, then $d_1 \leq -d(s_0, s')$. In particular, for $d_1 = \min(d - d(s, s_0), d'' - d(s_0, s''))$, we conclude that

$$\min(d - d(s, s_0), d'' - d(s_0, s'')) \leq -d(s_0, s'). \tag{8}$$

The minimum in the left-hand side of the inequality (8) is equal either to the first, or to the second term.

If this minimum is equal to the first term, then from (8), we can conclude that $d - d(s, s_0) \leq -d(s_0, s')$, i.e., that $d + d(s_0, s') \leq d(s, s_0)$. Since $d = d(s, s')$, we can reformulate this inequality as

$$d(s, s') + d(s', s_0) \leq d(s, s_0). \tag{9}$$

On the other hand, triangle inequality means that

$$d(s,s')+d(s',s_0)\ge d(s,s_0). \tag{10}$$

Comparing (9) and (10), we conclude that $d(s,s')+d(s',s_0)=d(s,s_0)$, i.e., that s' is in between s and s_0.

Similarly, if the minimum in the left-hand side is attained at the second term, s' is in between s_0 and s''. In both case, 1-D property is proven.

So, if $R\times S$ is a lattice, S is convex and acyclic 1-D.

Second part of the proof. Let's now prove that if S is convex and acyclic 1-dimensional, then $R\times S$ is a lattice, i.e., that for every two events (t,s) and (t'',s'') from $R\times S$ there exist the union and the intersection.

We will only prove the existence of the intersection; for the union, the proof is similar.

If one of the two given events precedes another one, then this preceding events is the desired intersection. So, it is sufficient to consider the case when neither of the two events precedes another event, i.e., the cases when $s-s''<d(s,s'')$ and $s''-s<d(s,s'')$; in other terms, the cases when $|s-s''|\le d(s,s'')$.

Let us construct the desired intersection. From the first part of the proof, we know that if (t',s') is the intersection, then s' is in between s and s'', and $c(t-t')+c(t''-t')=d(s,s'')$. Hence,

$$t'=\frac{ct+ct''-d(s,s'')}{2c,} \tag{11}$$

and s' is a point at a distance

$$d=c(t-t')=\frac{ct-ct''+d(s,s'')}{2} \tag{12}$$

from s and at a distance

$$d'' = c(t'' - t') = \frac{ct'' - ct + d(s, s'')}{2} \tag{13}$$

from s''.

Let us use this conclusion to construct the intersection. Namely, we will compute t' according to the formula (11), and take as s' the point in between s and s'' that is at the distance d from s and at the distance d'' from s''. Let us show that the corresponding event (t', s') is indeed the intersection.

Let's first prove that $(t', s') \preceq (t, s)$. Indeed, this requirement is equivalent to $c(t - t') \geq d(s, s')$, but accordung to our choice of s', we have $c(t - t') = d(s, s')$. Similarly, one can prove that $(t', s') \preceq (t'', s'')$. So, (ct', s') indeed precedes both events (t, s) and (t'', s'').

To complete the proof, we must now show that this event (t', s') is the *largest* of all events that precede (t, s) and (t'', s''). Indeed, let $(t_1, s_1) \preceq (t, s)$ and $(t_1, s_1) \preceq (t'', s'')$ (i.e.,

$$ct_1 \leq ct - d(s, s_1) \tag{14}$$

and $ct_1 \leq ct'' - d(s'', s_1)$). Let us show that $(t_1, s_1) \preceq (t', s')$, i.e., that

$$ct_1 \leq ct' - d(s', s_1). \tag{15}$$

Since the space S is acyclic 1-D, either s' is in between s and s_1, or s' is in between s'' and s_1. Without loss of generality, let us consider the first case. In this case,

$$d(s, s_1) = d(s, s') + d(s', s_1). \tag{16}$$

Substituting the right-hand side of (16) instead of $d(s, s_1)$ into the formula (14), we conclude that

$$ct_1 \leq ct - d(s, s') - d(s', s_1). \tag{17}$$

According to our choice of s', $c(t-t')=d(s,s')$, so, $ct-d(s,s')=ct'$, and the inequality (17) turns into the desired inequality (15). Q.E.D.

5. OPEN PROBLEMS

We have shown that the *major idea* of quark confinement can be reformulated in geometric terms. It is desirable to deduce the *quantitative formulas* for quark confinement from the geometric model. In particular, it is desirable to relate the physical properties of multi-quark particles (mesons and barions) with the geometric location of several quarks, and hopefully, explain the empirical upper limit on the number of quarks in a particle by geometrical combinatoric properties of quark location.

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