Intelligent Control in Space Exploration: Interval Computations are Needed

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***Abstract*— This paper is a short overview of our NASA-supported research into the necessity of using interval-based intelligent control techniques for space exploration. This is a reasonably new application area for interval computations, and we hope that the overviewed results and problems will be of some interest to the numerical computations community.**

This paper is a short version of our detailed report presented to NASA.

I. Intelligent Control is Necessary for Space Exploration

Control is necessary for space missions. For a space mission to be successful, it is vitally important to have a good control strategy for all possible situations. For example:

- For a *Space Shuttle*, it is necessary to guarantee the success and smoothness of docking, the smoothness and fuel efficiency of trajectory control, etc.
- For an *automated planet mission*, e.g., for a rover mission to Mars, it is important to control the spaceship's trajectory, and after that, to control the rover so that it would be operable for the longest possible period of time.

It is often difficult or impossible to apply methods of traditional control theory. In many complicated control situations, in particular, in many control situations related to space flights, methods of traditional control theory are difficult or even impossible to apply. The main reason for that difficulty is as follows:

- For *traditional control*, we must *know* (more or less *precisely*) the properties of the controlled system.
- However, *space missions* are usually sent to explore new phenomena, and must operate under extreme conditions. Therefore, our prior *knowledge* about the situation is *not complete.*

Even when we *do* know the system precisely, this description may be so complicated that computing the optimal control is computationally intractable [85].

Intelligent control is needed. In general, in uncertain situations, where no routine methods are directly applicable, we must rely on the creativity and skill of the human operators. And, indeed, expert controllers are very good in controlling complicated processes (not only in space missions, but also in the chemical industry, in metallurgy, in business). These experts usually cannot explain their control strategy in precise mathematical terms, but they can describe their strategies in terms of natural language, by phrases like "If a Space Station is close, and the relative speed is medium, decelerate a little bit". So, in order to develop an *automated* controller,

we must somehow transform these informal rules into a precise control strategy.

The methodology of such transformations is called *intelligent control.*

Intelligent control is useful also for *non-automatic* control: For example, for manned space missions, there are astronauts who are the best in solving the control problems. We cannot clone the best controllers, but we want to have an automated device that would simulate the best experts, and thus help other astronauts to control the mission.

Intelligent control is possible. There exist several dozens of different methods for intelligent control. This activity started in the 1970's by L. Zadeh and Mamdani, and has now many important applications, ranging from the Japanese automated subway system to various appliances.

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II. The choice of intelligent control methodology can drastically change the quality of the resulting control

The experience in applying intelligent control shows that sometimes a transformation method leads to unstable or non-smooth control. In such situations, a different transformation of rules into control would be more appropriate.

Usually the choice of an appropriate technique is made on a trial-and-error basis.

For space applications, it is necessary to have theoretical methods for selecting the best intelligent control methodology. Experiments performed at Johnson Space Center on the Shuttle and rover simulators, showed that intelligent control techniques really lead to high quality control of space missions and planet rovers.

However, most of the existing intelligent control techniques are semi-*heuristic*, in the sense that they rely partly on trial-and-error. This may be acceptable for an appliance, but it is definitely unacceptable to choose a technique on a trial-and-error basis for a billion-dollar project. So, we need *guaranteed* (theoretical) methods to choose an appropriate technique.

So, we must:

- *analyze the existing* intelligent control *techniques*, and
- *find out which* of these techniques *is the best* with respect to the basic optimality criteria: stability, smoothness, robustness, etc.;
- if for some problems, none of the existing techniques is of satisfactory quality, *design new, better intelligent control techniques.*

This paper is an overview. The results of our research are described in detail in the papers [1]–[91] and in the student theses and projects supported by this project [T1]–[T6]. In this paper, we give a brief overview of these results.

Before we formulate these results, we will briefly remind the reader of the main features of the most widely used existing intelligent control methodology: fuzzy control.

III. Fuzzy control: in brief

Rules. Fuzzy control methodology starts with expert "if-then" rules, i.e., with rules of the following type:

If x_1 is A_1^j and x_2 is A_2^j and $\ldots$ and x_n is A_n^j, then u is B^j,

where x_i are parameters that characterize the plant, u is the control, and A_i^j, B^j are the natural language terms that are used to describe the j^{th} rule (e.g., "small", "medium", etc).

Mamdani's transformation. The value u is a proper value for the control if and only if one of these rules is applicable.

Thus, the property "u is a proper control" (which we will denote by $C(u)$), can be, therefore, described as follows:

$$C(u) \equiv (A_1^1(x_1)\,\&\,A_2^1(x_2)\,\&\,\ldots\,\&\,A_n^1(x_n)\,\&\,B^1(u))\vee$$
$$(A_1^2(x_1)\,\&\,A_2^2(x_2)\,\&\,\ldots\,\&\,A_n^2(x_n)\,\&\,B^2(u))\vee$$
$$\ldots$$
$$(A_1^K(x_1)\,\&\,A_2^K(x_2)\,\&\,\ldots\,\&\,A_n^K(x_n)\,\&\,B^K(u))$$

Membership function. The natural language terms are described by *membership functions*, i.e., we interpret $A_i^j(x)$ as $\mu_i^j(x)$, the number (called *degree of belief*) that describes the expert's degree of belief that a given value x satisfies the property A_i^j. Similarly, $B^j(u)$ is represented as $\mu_j(u)$.

"And" and "or" operations. The logical connectives & and $\vee$ are interpreted, in this context, as operations $f_\&$ and $f_\vee$ on degrees of belief. The most frequent choices of these operations are $\min(a,b)$ and $a\cdot b$ for $f_\&(a,b)$, and $\max(a,b)$ and $a+b-a\cdot b$ for $f_\vee(a,b)$.

After these interpretations, we can form the membership function for control:

$$\mu_C(u) = f_\vee(p_1,...,p_K),$$

where

$$p_j = f_\&(\mu_{j,1}(x_1),\mu_{j,2}(x_2),...,\mu_{j,n}(x_n),\mu_j(u))).$$

Defuzzification. The system must supply a control, so we must end up with a single value u of the control that will actually be applied. An operation that transforms a membership function into a single value is called a *defuzzification.* To complete the fuzzy control methodology, therefore, we must apply some defuzzification operator F to the membership function $\mu_C(u)$ and thus obtain the desired value $\bar{u} = f_C(\vec{x})$ of the control that corresponds to $\vec{x} = (x_1,...,x_n)$. The most widely used defuzzification procedure is *centroid defuzzification*

$$\bar{u} = \frac{\int u\cdot\mu_C(u)\,du}{\int \mu_C(u)\,du}.$$

IV. Preliminary Research: Is the Existing Intelligent Control Methodology Reasonable? Universally Applicable? Optimal?

Before we start fine-tuning the existing intelligent control techniques, we must first make sure that this methodology is indeed good, i.e.,

- that this methodology is *reasonable*, i.e., consistent both with common sense and with other successful formalisms proposed to represent human reasoning;
- that this methodology is *universally applicable*, i.e., in principle, it can be used for an arbitrary control situation; and finally,
- that this methodology is indeed *optimal* in some reasonable sense, i.e., it can, potentially, lead to the best possible control.

In our preliminary research, we have shown that this is indeed the case.

A. Fuzzy control is reasonable

Let us first show that fuzzy logic is indeed consistent with other formalisms proposed to describe commonsense reasoning. All these formalisms can be viewed as modifications of *classical* (2-valued) *logic*, the logic that describes the ideal reasoning. Both in classical logic and in its commonsense modifications, we start with elementary (atomic) statements and combine them by using logical connectives (such as "and", "or", "not") and quantifiers (such as "for all" and "there exists") into more complicated logical statements.

The classical 2-valued logic can be characterized by the following features:

- in classical logic, every elementary statement is *either true, or false*;
- these statements can be combined by basic *logical connectives* and *quantifiers*;
- the truth value of the resulting complicated statements is determined by the *rules of logic*: e.g., "for all x, $A(x)$" is true if and only if the statement $A(x)$ is indeed true for all x.

At first glance, these features sounds perfectly reasonable. However, in real life, our reasoning does not always follow these rules:

- First of all, in real life, we are often *not sure* whether a certain statement is *true or false.*
  To describe this "un-sureness", it is desirable, in addition to the classical truth values "true" and "false", to have *intermediate degrees of belief.* If we add such degrees of belief, we get a modification of a classical logic that is called a *multiple-valued logic*:
  - Fuzzy logic is one particular case of this logic, in which we assume that there are *infinitely* many different degrees of belief that fill the entire interval $[0, 1]$.
  - Other multiple-valued logics, with *finitely* many different degrees of belief, have also been proposed to describe commonsense reasoning.

  Comment. To a certain extent, the third value — "unknown" — is already present in the classical logic system, in the sense that in a formal system, due to Gödel's theorem, a statement S can not only be *true* (when S is deducible from the theory) or *false* (when its negation $\neg S$ is deducible from a theory), but it can also be *unknown*, when neither the statement S itself, nor its negation can be deduced from the theory. In this sense, multiple-valued logics are not so much *replacing* the traditional logic, but they are *enriching* these logics by providing a finer structure of this "unknown". In particular, in [74], we show that traditional "paradoxes" of fuzzy logic, like the possibility of a statement to be (to some extent) true and, at the same time, (to some extent) false have natural analogies in classical logic.
- Second, in commonsense reasoning, the meaning of *connectives* is sometimes slightly *different* from its meaning in classical logic.
  For example, in classical logic, if one of the atomic statements $A_1, \ldots, A_n$ is false, then the compound statement $A_1 \& \ldots \& A_n$ is false. Not so in commonsense reasoning. For example, if the objective of a Space Shuttle's mission was to investigate a new geophysical area (A_1), to experiment with the signal transmission (A_2), to repair a satellite (A_3), etc., and to launch a new satellite (A_n), and the mission succeeded in all but one parts of this mission, then,
  - according to classical logic, we must say that the mission has failed, while
  - from the commonsense viewpoint, this mission was highly successful.

  There are crucial situations where all goals must be satisfied. Depending on the situation, the same word "and" can mean different operations. So, another approach to formalizing commonsense reasoning is to replace a *single*, say, "and" operation with *several* different operations that describe different commonsense meanings of "and".
  For better understanding, the difference between the classical logic and this new approach

can be illustrated *graphically*: if we represent each statement by a point, then,

 – in classical logic, the combined statement "A and B" is well defined and thus, also represents a *point*, while

 – in this new approach, we get the whole *line* of different values depending on which interpretation of "and" we choose. In view of this interpretation, this approach is called *linear logic*.

- Finally, formulas of *classical logic* are known to be, in general, *algorithmically undecidable* in the sense that there is no algorithm for finding the truth values of all composite logical statements.
  Thus, in commonsense reasoning, when we want to estimate the truth values of these statements, we have to use some *heuristic* algorithmic techniques that, in general, only *approximate* the actual (non-algorithmic) truth values of these statements.
  The corresponding approach to commonsense reasoning, in which we make *logical* reasoning algorithmic (i.e., implementable by a *program*), is called *logic programming.*

All three approaches turned out to be consistent with fuzzy logic:

- Formulas stemming from the *finite-valued logic* turn out to be exactly the formulas of *fuzzy logic* and fuzzy systems [76].
- Formalisms of *fuzzy* and *linear* logic are so close that we can justifiably call fuzzy logic "applied linear logic" [54, 75].
- Finally, logical equivalence stemming from *logic programming* turns out to be equivalent to the one that comes from *fuzzy logic* [68, 71, 72].

In all three cases, we not only show that fuzzy logic is *consistent* with the other formalisms, but we get a new *justification* of previously heuristic methods and formulas of fuzzy logic in terms of these other formalisms:

- *Finite-valued logic* helps to justify certain *and* and or operations of fuzzy logic (namely, min and max), the extension principle, fuzzy optimization, etc. [76].
- Axioms of *linear logic* justify the general properties of fuzzy *and* and *or* operations, such as associativity [54, 75].
- Finally, *logic programming* explains why in the most successful approach to fuzzy control — Mamdani's approach — implication is (weirdly) interpreted as "and" [30, 72].

B. Fuzzy control is universally applicable

Main result: fuzzy control is a universal approximation for (crisp) control strategies. It has been known that fuzzy control is a universal methodology for *traditional* control problems, with *one or several inputs*. To be more precise, it has been known that an arbitrary control can be, within an arbitrary accuracy, approximated by an appropriate fuzzy controller.

In [52, 79], this result is extended to *distributed* systems in which the state is described by a function, and to even more general control situations.

Auxiliary result: "fuzzy control"-type statements are a universal approximation for arbitrary fuzzy statements about control. The universality results mentioned above mean, in particular, that if we know a *crisp* (non-fuzzy) control strategy, then we can have a set of fuzzy control rules that approximate this strategy with any given accuracy.

In real life, however, we do not know this crisp strategy; instead, we have a (*fuzzy*) expert *knowledge* about it. In some cases, this knowledge is already formulated in terms of if-then fuzzy rules; in fuzzy control methodology, these rules are transformed into statements that only use connectives "and", "or", and "not".

However, in many other real-life cases, the fuzzy knowledge about control can be of much more general type. Therefore, the question appears: can we approximation an arbitrary fuzzy knowledge, that uses *arbitrary* logical *connectives* (including different versions of fuzzy implication), by a knowledge described in terms of "and", "or", and "not" fuzzy connectives? In other word, can we approximate an arbitrary fuzzy logical connective by a combination of these three basic ones?

It may seem, at first glance, that different unusual connectives are purely mathematical constructions, but, as we show in [87], even those connectives that may appear this way actually result from very natural axioms. In view of this result, it is desirable to consider the approximability of *arbitrary* logical connectives.

It turned out [77, 80] that in general, such an approximation of an arbitrary connective *is* possible, but only when we, in addition to these three basic connectives, allow *modifiers* such as "very", "slightly", etc. (that without the modifiers, such an approximation is impossible, is also shown in [53]).

C. Fuzzy control is optimal

Main result: fuzzy control is optimal (in some reasonable sense). In many space-related problems, we need the control results really fast; in these situations, to speed up the computations, it is natural to use *several processors* working *in parallel.* In [63], we have shown that if we want the *fastest* possible parallel *universal* computer (i.e., a computer with the ability to approximate an arbitrary function), then we get an architecture that corresponds exactly to *fuzzy control* methodology.

Thus, fuzzy control methodology is indeed, in the above sense, *optimal.*

Fuzzy control and neural control: which is the best? The paper [63] also contains a comparison between the fuzzy control methodology and another widely spread area of intelligent control: *neural network* control. Namely:

- If we consider *digital* processors, then *fuzzy control* is the optimal methodology.
- However, if we consider *analog* processors, then the same optimality criterion leads to the selection of *neural network control* methodology.

On one hand, neural network control is somewhat similar to the fuzzy control:

- the description of a fuzzy controller consists of elementary objects *rules*;
- the description of a neural network controller consists of elementary objects: *neurons.*

This similarity shows itself in the fact that for a natural expert system application, both approaches lead to the same class of methods [38].

On the other hand, this analogy is not complete, there are major differences in these two methodologies:

- In fuzzy control, *rules* (i.e., the corresponding elementary objects) come *directly* from the experts.
- On the other hand, for neural network control, *neurons* and their weights usually come from a lengthy and extremely time-consuming training.

This specific neural problem of determining the weights of the neurons naturally leads to the following two questions:

- First of all, how uniquely are these weights determined by the control that we are trying to approximate? The answer to this question — "yes, uniquely" — is given in [82].
- Second, how many neurons do we need to approximate a given control? In general, we may need a lot, but in [58, 59], a general class of controls is described for which the number of necessary neurons remains quite feasible.

D. *The Optimal Choice of the Fuzzy Control Techniques*

Criteria for choosing a control. What do we want of the control?

- First, the control must *control.* In other words, if some external force has shifted the controlled object from the desired trajectory, then the system must return to the desired trajectory as soon as possible. In control theory, this property is called *stability.*
- Second, the control must lead to a *smooth* trajectory. *Smoothness* is extremely important both for manned and for automated space missions, because abrupt accelerations can be very uncomfortable to human astronauts and damaging for the sensitive equipment.
- The input data for control usually comes from sensors, and sensors are not 100% accurate. As a result, the measured values of the input variables that are used by the controller may be different from the actual values of the measured quantities. The ideal control must, therefore, work well not only for the input values, but also for the values that are close to the input ones. In other words, the uncertainty in the final control value, that is caused by the uncertainty of the input data, must be the smallest possible. Such controls are called *robust.*
- Finally, we want the computations of the control value to be as fast as possible. This computation speed is important for many control situations, but it is especially important for space missions, where decisions often need to be made in no time. As a result, in such situations, we must select the fastest possible algorithms, i.e., in computer science terms, algorithms with the *smallest possible computational complexity.*

Ideally, we would like to have a control that is the best according to *all* of these criteria, but in reality, these criteria are often conflicting with each other: e.g., if we want the system to be the *stablest* possible, i.e., return to the original trajectory as fast as possible, then a small deviation would result in a fast jerk back, making the trajectory *non-smooth.* In different situations, different criteria are most appropriate:

- For example, when we *dock* a Space Shuttle to a Space Station, the main criterion is *smoothness*, because non-smooth docking can seriously damage both the Space Shuttle and the Space Station.
- When we *track* a satellite by a radio signal, then our main goal is *not to lose* it; in this case, if we have accidentally deviated from the satellite's

position, we want to get the signal back as soon as possible. In terms of our criteria, in this situation, the main criterion is *stability.*

In this report, we describe which techniques are the best w.r.t. these basic criteria.

Comment. In addition to the situations where one of the above-described criteria is the most appropriate, we may have more complicated situations in which the objective function is the result of a *trade-off* between different criteria. A critical survey of different methods of optimizing a (crisp) criterion under (possible fuzzy) constraints was published in [8, 9, 10, 39].

Since we can have (potentially) infinitely many different combinations of criteria, we cannot explicitly describe the best control for all possible combinations. However, we hope that the general methods developed and used in this project can help in these more complicated situations as well.

E. How we solve the corresponding optimization problems

We must optimize under uncertainty. Fuzzy control is mainly used in situations when we do not have a complete knowledge about the controlled system; in other words, fuzzy control is mainly used in the presence of *uncertainty.* Hence, the problems of choosing the best technique (that we are interested in solving) are particular cases of *optimization under uncertainty.*

Optimization under uncertainty also occurs in fundamental physics. Choosing the best control technique is not the only real-life area where we must make conclusions in case of strong uncertainty. Strong uncertainty is also present in *fundamental* research, i.e., research in the areas where we have just started collecting data.

To handle such situations, theoretical physics has developed many useful approaches. One of these approaches that turned out to be one of the most successful in theoretical physics, is the so-called *group-theoretic* (*symmetry*) approach.

Group-theoretic (symmetry) approach in physics. It is well known that if a problem has a certain symmetry, then solving this problem becomes a much simpler task.

For example, if we are looking for a gravitational potential $\varphi(t, x_1, x_2, x_3)$ generated by several moving celestial bodies, then we must find a function of four variables by solving the corresponding partial differential equation. If, however, we know that there is only one body, and that this body is stationary and spherically symmetric, then we have two reasonable symmetries: invariance w.r.t. shift in time $t \to t + t_0$) and invariance w.r.t. rotations. In this situation, we must look for a solution that also has similar symmetries. If a function φ is invariant w.r.t. shift in time, this means that it does not depend on time at all. If a function is invariant w.r.t. rotations around the body's center O, this means that φ depends only on one variable: the distance r from a given point to the point O. Thus, the function φ turns into a function of 1 variable only: $\varphi(t, x_1, x_2, x_3) = \varphi(r)$, and a difficult-to-solve partial differential equation runs into a (much easier to solve) ordinary differential equation.

This idea of symmetries is used in physics not only to find solutions, but also to describe fundamental physical theories, the equations of most of which can be uniquely determined by the corresponding invariance requirements. This trend started with *special relativity* theory, whose main postulate was the postulate of *relativity*, i.e., invariance w.r.t. constant-speed motion. The notion of symmetry is so widespread that new physical theories are often formulated not in terms of different equations, but in terms of the corresponding symmetries.

Since symmetries are such a useful tool in physics, we want to use them for our problems as well.

Group-theoretic (symmetry) approach can be also used for selecting the best fuzzy control technique. In order to apply the ideas of symmetry to our problems, we must find out what the symmetries are in these problems.

There is a very natural symmetry here: namely, the very "fuzziness" of assigning *crisp* numbers to different "fuzzy" expert's degrees of belief means that different assignment procedures can be equally adequate. It is therefore natural to require the results of our processing the membership values (i.e., processing the results of this assignment) should not depend on which of the several possible equally adequate assignment procedures we choose. In other words, our processing algorithms must be invariant w.r.t *re-scaling*, i.e., w.r.t. moving from one scale of membership values to another possible scale.

It turns out that a natural formalization of this invariance can indeed solve the original optimization problems [8, 9]:

F. Results: the list of optimal methods

Optimal methods w.r.t. major optimality criteria. Part I. Choice of membership functions. The most *robust* membership functions are piecewise-linear ones [72, 78].

This result explains why the piecewise-linear membership functions are, at present, most frequently used.

Optimal methods w.r.t. major optimality criteria. Part II. Choice of "and" and "or" operations. (These results are (mainly) summarized in [8, 9, 72, 78].)

- If we are looking for the *most stable* control, then the best choice is to use $f_\&(a,b) = \min(a,b)$ and $f_\vee(a,b) = a + b - a \cdot b$ [53].
- If we are looking for the *smoothest* control, then the best choice is to use $f_\&(a,b) = a \cdot b$ and $f_\vee(a,b) = \min(a,b)$.
- If we are looking for the control that is *most robust*, then, depending on what we are looking for, we can get two different results:
  - if we are looking for the control that is the most robust *in the the worst case*, then the best choice is to use $f_\&(a,b) = \min(a,b)$ and $f_\vee(a,b) = \max(a,b)$ [72, 78];
  - if we are looking for the control that is the most robust *in the average*, then the best choice is to use $f_\&(a,b) = a \cdot b$ and $f_\vee(a,b) = a + b - a \cdot b$ [72, 78];
  - instead of minimizing the *average* error, we can try to minimize the corresponding *entropy* [39, 53]:
    * if we use the *average* entropy (in some reasonable sense), we get the same pair of optimal functions as for average error;
    * for an appropriately defined *worst-case* entropy (see also [83]) the optimal operations are $f_\&(a,b) = \min(a,b)$ and $f_\vee(a,b) = a + b - a \cdot b$.
- Finally, if we are looking for the control that is the *fastest to compute*, then the best choice is to use $f_\&(a,b) = \min(a,b)$ and $f_\vee(a,b) = \max(a,b)$.

Optimal methods w.r.t. major optimality criteria. Part III. Choice of defuzzification. In [39, 53], we show that the optimal defuzzification is given by the centroid formula.

Optimal methods w.r.t. additional optimality criteria: robustness w.r.t. possible computer malfunctions. *Robustness* can also mean robustness w.r.t. possible computer malfunctions. In principle, there are two possible types of malfunctioning:

- It can be a *temporary* malfunction, so all we need to do is *undo* the faulty operation and start again.
  In this case, we would like to have algorithms that make this "undoing" the easiest. In [6], we show that the possibility to undo is always present if and only if all membership functions are *fuzzy numbers*, i.e., if they have the simplest possible monotonicity structure (namely, they first increase, and then decrease).
- It can also be a *serious* malfunction, after which, for a certain period of time, further computations are impossible. In this case, we would like to have control implemented by an *interruptible* algorithm, i.e., by an algorithm that, if interrupted in the middle of the computations, still produces a reasonable control. In [3], it was shown that it is possible to transform every algorithm into an interruptible one without making its computation time much worse.

Optimal methods w.r.t. additional optimality criteria: optimal tuning in adaptive control. Similar optimization techniques have been applied to show that certain (fractionally linear) tuning formulas are the best in *adaptive* fuzzy control [T4].

Multi-criteria optimization. So far, we have considered situations in which we have a well-defined optimality criterion. However, in real life, we often have several *conflicting* criteria, especially when different participants of a project have slightly different aims. Optimization methods for such conflict situations are considered in [39].

V. A New (Interval) Approach to Fuzzy Control

The main goal of this project was not only to choose the *best* of the *existing* fuzzy control techniques, but also, if possible, to design *new*, better techniques.

To describe these methods, let us first explain why the traditional fuzzy control techniques are not always the most adequate.

A. Why intervals? Reason I: Intervals naturally appear

- Traditional fuzzy control techniques start with the expert's degree of belief that are represented by *numbers* from the interval $[0,1]$.
  - This use of numbers may be natural when we describe physical quantities, for which there exists a true value that can be, in principle, measured with greater and greater accuracy.

  - However, for degrees of belief, numbers may not be the most adequate representation.

Indeed, how are the existing *knowledge elicitation* techniques determine these numbers?

  - One of the possible techniques is to ask an expert to estimate his or her degree of belief by a number on a scale, say, from 0 to 10. Then, when an expert estimates this degree of belief by choosing, say, 6, we take $6/10 = 0.6$ as the numerical expression of the expert's degree of belief.

    At first glance, this may sound like a reasonable assignment, but in reality, the fact that an expert has chosen 6 does not necessarily mean that the expert's degree of belief is *exactly* equal to 0.6; it rather means that this degree of belief is closer to 0.6 than to the other values between which we have asked the expert to choose (i.e., to $0, 0.1, \ldots, 0.5, 0.7, \ldots, 0.9, 1.0$). Mathematically, values that are the closest to 0.6 form an interval $[0.55, 0.65]$. In other words, the only thing that we can conclude based on this choice is that the expert's *true* degree of belief belongs to the interval $[0.55, 0.65]$.

    In principle, we could try to get a more precise value of the degree of belief by asking an expert for a value on, say, a scale from 0 to 100, but hardly anyone can distinguish between degree of belief that correspond to, say, 63 and 64 on this scale. Thus, the interval $[0.55, 0.65]$ is the best we can get.
  - Another way of determining the degree of belief is to *poll* experts. If 6 experts out of 10 believe that, say, a given value of x is small, then we take $6/10 = 0.6$ as the degree of belief $\mu_{\rm small}(x)$ that this value x is small.

    Polls have their own margins of uncertainty. Hence, from a poll, we cannot extract the *exact* ratio of experts who believe that x is small; we can, at best, find an *interval* of possible values of this ratio.

    In principle, to get a narrower interval, we can ask more and more experts, but in reality, the number of experts is often limited, and asking all of them is not practically possible. As a result, the interval of possible values is the best we can get.

Similar conclusions can be obtained for all other methods of eliciting the values. For all these methods, an *interval* is a much more adequate description of the expert's degree of belief.

- Even if we manage to get narrow enough intervals for degrees of belief, so that these original degrees of belief can be adequately described as numbers, in the fuzzy control methodology, we need to *process* these numbers. The first processing consists of applying *and* and *or* operations.

  These operations, in their turn, must also be elicited from an expert so that they would be most adequate in describing what the experts mean when they use the corresponding connectives. We have already seen that eliciting *numbers* leads, in reality, to intervals. The resulting uncertainty is even worse if we try to elicit not a *single* number, but *several* different numbers that describe the desired *functions* $f_\&(a, b)$ and $f_\vee(a, b)$. As a result, instead of a *single* pair of functions, we, most probably, will get an *interval* of possible functions. If we apply this *interval* function to numerical input values, we get an *interval* of possible results.

  Thus, even if we managed to avoid intervals on the first stage, they will appear on the second stage of fuzzy control methodology: when we combine the original degrees of belief into degrees of belief of different rules.
- Even if we fix *and* and *or* operations, for the same query, we can have *different* representations in terms of "and", "or", and "not". These different representations are equivalent in classical logic, but in fuzzy logic, they are not. As a result, depending on which representation we use, we may get different numerical answers to the query. Hence, if we only know the query itself, and we are not sure what "translation" into basic logical operations is the best, it is natural to return not a *single* numerical value, but the entire *interval* of possible values of degrees of belief that correspond to different possible translations.

  In [91], we show how to compute this interval for different queries.

Comments.

- These three arguments (also given in [22, 31, 50, 69, 72]) do not exhaust all arguments in favor of intervals as a better way to describe uncertainty. Other arguments showing that two numbers represent uncertainty better are given, e.g., in [62].
- Intervals are very natural not only in the contents of fuzzy control, but also in computing in general. It suffices to say that:

  - actually, the modern calculus started with interval computations [13], and
  - that intervals are the only sets whose use preserves the invertibility of arithmetic operations [29].
- So far, we have only said that intervals are a *more adequate* tool for describing expert knowledge. This, in itself, does not necessarily mean that fuzzy control that comes from using intervals is in any sense *better*. However, it is reasonable to expect that more adequate description of expert's knowledge leads to the fuzzy control that more adequately describes expert's high-quality control and is, therefore, of a better quality itself. In the next section, we will show that these expectations were correct: *interval control is*, indeed, in many cases *better*.

B. Why intervals? Reason II: Intervals lead to a better control

Interval-valued fuzzy control. If we use *intervals* of possible values of initial degrees of belief, then, on all further stages of fuzzy control methodology, we also have to use *intervals* of possible values.

General idea behind using intervals in fuzzy control ([50, 64])**.** The main idea of using an *interval* to describe the expert's degree of belief, instead of more traditional technique of picking *a* number from this interval, is that the *actual* (unknown) degree of belief is *guaranteed* to belong to the interval, but it may be *different* from the picked value.

From this idea, one can (informally) conclude that the resulting interval control is often better than the original number-valued control. It turned out that this indeed true [50, 62]:

Intervals lead to a more stable control. Traditional fuzzy control techniques, if used appropriately, lead to a control that is stable.

- However, if we use only a *single* picked value from the interval of possible values, we will get a control that is *stable for this* particular *value*, and may *not* be *stable* at all for the *actual* value.
- The only way to guarantee that the control is stable for the *actual* (unknown) value is to guarantee that it is stable for *all* values from this interval. This requires, at least, that the algorithm that computes the control values should have this interval at its disposal.

Thus, to improve stability, we must have an algorithm that processes *intervals* of degrees of belief (rather than picked numerical values).

Intervals lead to a smoother control. A picked value of degree of belief is, in general, unpredictably ("randomly") different from the actual value. As a result, the control $\bar{u}$ coming from the picked values, will "wobble" around the control that correspond to the actual (unknown) degrees of belief. The random wobbling around a smooth process usually makes it less smooth.

Thus, the way to avoid this wobbling (and to make control smoother) is to take into consideration that the actual values are within the *intervals*, and then, to choose the *smoothest* possible control within these intervals.

Intervals lead to a more robust control. The control that takes into consideration the possibility of slightly different inputs is, by definition, *more robust* than the control that is based only on the original picked values.

Intervals sometimes lead to a computationally faster control. In general, computational simplicity is not the strongest point of interval computations (see below); however, in some situations, intervals do make computations faster.

Indeed, if we only have *numbers*, without any indication how accurate these numbers are, then, in order to guarantee the accuracy the resulting computations, we have to do all the data processing with all the digits of all these numbers.

If we know *intervals* instead of numbers, this means, in essence, that we know the *accuracy* of the input values. If the input values are known, say, with accuracy of 10%, then there is no much sense to have computations with much better accuracy, so we can use *fewer digits* in our computations and thus, make these *computations* much *faster*.

C. How to elicit interval membership functions?

In order to apply fuzzy control methodology, we must first elicit the membership functions and "and" and "or" operations that best describe the expert (or experts) whose opinions we are formalizing.

For this elicitation, we can use two sources of information:

- First, we can *interview* experts and try to extract the required information from their answers. For interval-valued degrees of belief, the corresponding problem is formulated and partially solved in [11].
- Interview is an ideal method, but often, experts who are very good in controlling are not that

good in the ability to describe their control in words. Actually, the very necessity of fuzzy control comes from the fact that experts are not very good in describing their control strategies. For such experts, an important source of membership functions and other information is their actual control: we can

- simulate different situations;
- record how these experts would control the desired object, and then
- try to extract their membership functions and other information from these records.

In [7], we show that it is always possible to extract this information from the records, and we describe how exactly this can be done.

D. The main problem of interval approach — computational complexity — and possible solutions of this problem

The problem. We have mentioned that the use of intervals often *improves the quality of* intelligent *control*. However, an apparent *disadvantage* of their use is that when we consider interval-valued instead of more traditional number-valued degrees of belief, we need to process twice as many numbers and therefore, the computational complexity (and thus, the computation time) increases. This increase can be very drastic: e.g.,

- While the solution of a linear systems $\sum a_{ij}x_j = b_i$ with crisp coefficients a_{ij} and b_i is a relatively easy problem, the solution of a linear system of equations with *interval* coefficients is, in general, computationally intractable (NP-hard) (see, e.g., [20, 44, 61]), even if we restrict ourselves to narrow intervals only [T5]. This computational complexity can be "explained" if we look at the geometric shape of the corresponding solution sets:
  - for crisp linear systems, the solution set is a *convex polytope*;
  - for interval linear systems with symmetric matrices $a_{ij} = a_{ji}$, the shape of the solution set becomes *piecewise-quadratic* [2];
  - for interval linear systems with dependent coefficients, we can have *arbitrarily complicated algebraic shapes* [1];
- Even when we have *explicit* computations (e.g., if we compute the value of a polynomial $f(x_1, \ldots, x_n)$) instead of solving systems of equations, for *interval-valued* inputs $x_1, \ldots, x_n$, the problem becomes NP-hard, and the shapes become algebraic shapes of arbitrary complexity [33, 43]. (In [40], a similar result is expressed in a slightly different form: if we want interval computations without roundoff errors, then we have to use algebraic numbers of arbitrary complexity.)
- For *expert systems* that use numerical degrees of belief, as soon as we have been able to express a given query as a logical combination of the statements from the system, computing the degree of belief in this query becomes a pretty straightforward and easy task. However, when we have *interval-valued* degrees of belief, the problem becomes NP-hard [T2].
- Closer to home, the problem of *eliciting* interval-valued *membership functions* is, in general, NP-hard [11].
- Also, the problem of finding the *optimal control* is, in general, NP-hard [85].
- Unfortunately, these results stay even if we consider a more realistic fuzzy-based formalization of feasibility [70, 72].

A general survey of such problems was given in [45] (see also [67]).

Comment. A similar trade-off between the control quality and its computational complexity can be observed if we compare *interval* methods with more traditional *statistical* methods:

- *interval methods* lead to *better estimates* [89], but
- *interval methods* are, in general, *more computationally complicated* [28].

How can we solve this problem? There are several possible ways to solve this problem:

- If we cannot find fast algorithms that work well in *all* cases, then we can look for algorithms that work well in *almost all* cases. In particular, for narrow intervals, the existence of such algorithms was shown in [60].
- If we cannot find an algorithm that works well in *almost all* cases, then, at least, we can try to look for specific cases in which fast interval algorithms are possible. In particular, we discovered such algorithms for the following problems that correspond to different stages of fuzzy control methodology:
  - for some "and" and "or" operations, the problem of *eliciting* the interval-valued membership functions becomes computationally feasible [11];
  - fast algorithms are also known for the case when the functions are *monotonic* [25]; *"and" and "or" operations* are usually monotonic;

– computing the range of *fractionally linear* functions [64]; this is important for applying *defuzzification*, which is usually described by a fractionally linear transformation;

– "smoothing" an interval function [57]; this is very important for designing a *smooth* control;

– locating local extrema of a function of one variable from interval measurement results [65, 88]; this is extremely important for *optimization*;

– finally, a fast algorithm is designed that checks *stability* of the resulting control [72].

- If we cannot find methods that are *guaranteed* to work well, then at least we may find *heuristic* methods that may often work. As part of this research, we have proposed and analyzed both the modifications of the existing heuristic methods, such as *genetic algorithms* [21] and *chemical computing* [38, T3], and proposed new interval-based heuristics [38, 86].
  For heuristic methods, two questions naturally arise:

  – We know that sometimes these methods do not lead to the best possible results. Can we, given the inputs, check whether this method will work or not? and how good the results are?

  – In many cases, heuristic methods contain several parameters that need to be tuned. Depending on how we choose the values of these parameters, we may get very good results or very lousy results. How can we choose the optimal values of these parameters?

  In this research, we attack both question:

  – In [49], we design a method for estimating the quality of interval computations. To be more precise, there exist several methods that compute the *enclosure* (superset) of the desired interval. Methods from [49] generate a *subset* of this interval. If the resulting two interval are close, this means that the enclosure is a good estimate of the desired interval.
    In a more general context, the *rating* of different methods is proposed and justified in [T6].

  – To find the optimal values of the parameters of heuristic methods, we use the general group-theoretic (symmetry) approach [56]. In particular, in [73], we show that re-scaling, a useful heuristic technique in fuzzy control and in genetic algorithms, should be best avoided in the case of complete uncertainty.

- If the interval-related *mathematical* problem that we are trying to solve is still too complicated, we may want to check whether this mathematical problem is indeed an adequate formalization of the original real-life problem.
  In many cases, as Zadeh himself mentions, the complexity of the model is caused by the fact that the model tries to describe the original *low-granularity* problem, with few distinct levels of a certain quantity (like "small", "medium", and "large") by a model in which this quantity is described by a real number and thus, has infinitely possible values (*high granularity*).
  Discovering that this indeed is the source of the problem is one thing; the next important step is to see what we can do in this situation to speed up computations. In [42], we describe how we can possibly do computations *directly* with low-granularity values, without translating them into high-granularity numbers.
- Finally, if we *cannot* think of any way of making an *algorithm faster*, we can still speed up the computations if we make *interval operations hardware supported* (and thus, faster).
  It is impossible to hardware support all possible operations with intervals. In view of that, in [51, 81], we analyze (and solve) the problem of choosing the interval operations whose hardware support will lead to the largest computation speed-up; the answer, crudely speaking, is as follows: in addition to interval analogs of standard arithmetic operations, we must support an operation of weighted dot (scalar) product $a_1, \ldots, a_n, b_1, \ldots, b_n \to \sum w_i \cdot a_i \cdot b_i$.

E. Additional method of improving fuzzy control

An additional method of improving the quality of fuzzy control was proposed in [19].

The main problem that this method is dealing with is that in traditional fuzzy control techniques, all rules are on equal standing. As a result, even when an expert explicitly says that for $x = 1$ the control should be exactly $u = 4$, the technique mixes this conclusion with other rules (like "when x is small, u should be small") and, as a result, returns the control $\bar{u}(1)$ that is often different from the desired $\bar{u} = 4$.

R. Yager and other researchers have proposed to remedy this situation by introducing the explicit *priorities* of different rules. In [19], we show that the same effect can be achieved without any additional information, simply by (slightly) modifying Mamdani's logical transformation.

A similar idea leads to a more adequate formalization of more complicated expert knowledge that includes *binary* properties like "x is approximately equal to y" [32].

VI. Auxiliary Results. Part I: Technical Diagnostics

Traditional fuzzy control techniques are designed mainly for the case when the controlled system functions well, and the question is only how to control it. In real life, however, and especially in space flights, malfunctions are quite possible. In this case, we have a problem of finding out which exactly component of the system is wrong.

If the system is simple and all its components are easily accessible, then we can simply test all its components. In space missions, however, systems are very complicated, and some components are difficult to access. As a result, we cannot simply test all the components, we need some intelligent algorithm to find the faulty component without testing all of them.

Similarly to fuzzy control, there are engineers who are very good in such a diagnosis, so it is natural to use their experience to diagnose the systems. Such technical diagnostic methods are developed for two possible types of malfunction:

- In [39], methods are described that find the faulty component for the case when the system stops functioning.
- In [4], methods are described that locate the faulty component in the situations when the system continues to function, but the value at least one of the critical parameters (that characterize the system's behavior) gets out of the interval of admissible values.

VII. Auxiliary Results. Part II: Applications to Space-Related Data Processing

A. Data Processing is Important

Computation of the optimal control strategy is not the only space-related computation. Indeed, why do we need to launch space missions in the first place? One of the main objectives of the space flights is to bring the *information* about objects and processes, both in space and on the Earth. This information rarely comes in the desired form, it usually requires some *processing*.

Computation of the best control strategy can also be viewed as a *particular case* of data processing: namely, we take as inputs the sensor data, and we return the desired control. It is therefore reasonable to try to apply the methods and results, that were originally designed for control-related data processing, to general space-related data processing.

B. Major Areas of Space-Related Data Processing

In order to describe how these ideas can be used in space-related data processing, let us first enumerate the major areas of space-related data processing:

- At present, most space missions occur in the close vicinity of the Earth, and all of them are in the Solar system. Thus, the major area of space-related data processing is the analysis of near-Earth environment from the results of data processing.
- The near-Earth environment is not the only area about which we learn more after the space missions. Space is also the area from where, undisturbed by the Earth atmosphere, we can:
  - clearly observe the distant bodies and thus, get a large-scale picture of our Galaxy and of the Universe as a whole;
  - precisely trace the effects of the gravitation and thus, get a very clear picture of the relativistic effects and, in general, of the space-time (in particular, Dr. Jorge Lopez from Physics Department of the University of Texas at El Paso is doing this data processing from JPL);
  - observe high-energy particles and processes and thus, get a clearer understanding of the fundamental physical processes.
- Last but not the least, space flights, especially near-Earth space flights, brings us a lot of *geophysical* information, i.e., information about our Earth. The importance of this application area is emphasized by the fact that the Mission to Planet Earth is one the main missions of NASA.

C. Near-Earth observations

For near-Earth observations, we can formulate the following three problems:

- First, we would like to *estimate* the *accuracy* of the existing indirect measuring techniques.
- Second, for the situations when the resulting accuracy is not sufficient, we would like to design *new*, more accurate indirect measurement (= data processing) methods.
- Traditional data processing results in *numbers* that still have to be analyzed. Therefore, it is desired, in addition to this traditional data processing, to have more *intelligent* data processing that would provide us directly with the

answers to the *fundamental* questions about the Solar system, questions that we are really interested in.

As part of the project, we solved the simplest cases of all these three problems:

Error estimation.

- Most of the instruments and sensors used in space missions are *similar to the instruments used on Earth*, and so, we can use the results of error estimation obtained in the analysis of Earth measurements (see, e.g., [5, 12, 18, 22, 31, 48]).
- There are, however, a few instruments and sensors that are *more specific* for space environment.

Namely, one of the main advantages of space observations is that in space, there is practically no atmosphere, and therefore, *optical observations* can be drastically more accurate than on Earth. This comment relates both:

- to *passive* observations, when we simply use an orbital telescope to observe the light coming from the celestial bodies, and
- to *active* observations, when we artificially "brighten" the objects and then observe the reflected light.

To "brighten" the images, we must use a very strong source of light; so far, the strongest sources of light are *lasers*, so, we arrive at the problem of *estimating accuracy for laser observations*.

A particular case of this problem was considered and solved in [84].

New data processing methods. As we have just mentioned, most near-Earth measurements are very similar to Earth measurements. There are, however, a few things that are radically different in space. The major difference is that:

- on Earth, all the matter is usually in one of the three main states: solid, liquid, and gas.
- In space, many substances are in the fourth state: of *plasma*, where, instead of electrically neutral atoms, we have *charged* particles: electrons and ions.

The abundance of charged particles often creates currents, magnetic fields, etc., that are much stronger than we are used to, and therefore, cannot be directly measured by means of traditional sensors. For these measurements, we need a new methodology.

In [55], we design a new method of measuring string current by measuring magnetic fields that these currents generate; so far, the algorithm is applied to the Earth situations in which strong current are artificially created: to the string currents used in aluminum production.

Fundamental questions about the solar system. So far, the Solar systems works as a clockwork; it looks like catastrophes are highly unprobable. However, the huge masses of celestial bodies, together with the high speeds, make every collision truly catastrophic. So, one of the most fundamental questions is: *Is the Solar system truly stable or a big collision is inevitable*?

This problem is very *difficult to solve numerically* because small numerical uncertainties (that are inevitable in calculations) increase exponentially and make the results of long-term numerical calculations useless for predictions. So, the only way to guarantee stability is to have predictions with *a guaranteed accuracy*.

In [41], we apply interval methods to the stability problem: namely, we show that within a certain reasonable hypothesis, our *Solar system is stable*.

D. Relativistic Effects and the Structure of Space-Time

For these applications, to measuring geometry of space-time, we have two types of results:

- First, we show that the corresponding problems are, in general, very *computationally complicated* [26]. Even when the corresponding problems are computationally feasible [27], the problems of measuring proper distances and proper times in space-time geometry are much more complicated than the problems of measuring distances in Euclidean space [35].
- Second, we show that the general *group-theoretic* methodology can be successfully applied to these problems.
  In particular, we show that reasonable axioms of space-time geometry that are usually formulated in geometric and causal terms can be reformulated in terms of symmetries [37]. This general reformulation turns out to be quite useful: e.g., causality explains the previously unexplained physical fact about symmetries: that spatial and temporal translations commute [36].

E. Fundamental Physical Processes

Modern physics is based on *quantum mechanics*, which is usually interpreted in *probabilistic* terms. At first glance, there seems to be no big need for using *fuzzy* and/or *interval* methods. However, a more attentive analysis reveals some fundamental problems in traditional probabilistic approach:

- First, the equations of quantum filed theory often lead to meaningless *infinities* instead of the physically meaningful finite values. There exist several semi-heuristic methods of handling these infinities, but it is definitely desirable to avoid them from the very beginning.
- For some possible physical processes that are seriously considered in modern physics (e.g., for acausal processes), the standard probability approach encounter problems (see, e.g., [14]).

To handle both problems, we first showed, in [24], that *standard quantum mechanics* approach can be viewed as a *particular case* of the more general *fuzzy* approach (of which interval uncertainty is another particular case), and that many supposedly specifically quantum phenomena can be thus explained [47] as pure mathematical consequences of the formalism rather than a necessity for a new specifically quantum approach. With this embedding, we have a more general formalism, and we show that both problems can be naturally handled within this more general formalism:

- In [23], we show that if we take into consideration *measurement uncertainty*, in particular, *interval uncertainty*, then the equations of physics become consistent.
- In [14], we show that the natural description of *acausal* processes leads to *non-probabilistic uncertainty.*

As a side effect of these results, in [46], we *explain* why the group-theoretic (*symmetry*) approach, an approach which has originated on physics and which we have so successfully used in our research, is useful in physics.

F. Mission to Planet Earth

Specific feature of geophysical data processing is that we also have lots of Earth data. The processing of geophysical data is one of the main areas of space-related data processing. In particular, this data processing is one of the main areas of the NASA Pan-American Center for Earth and Environmental Studies (PACES) that operates in El Paso, Texas.

The specific feature of this application area (as opposed to pure space research) is that, in addition to information coming from space flights, there is also *lots* of geophysical *information* about the same areas coming from the *Earth measurements.* It is therefore important to process *both* types of measurement results.

With new space data, a new problem arises: estimating accuracy of the results of data processing. Many data processing methods have been developed in traditional geophysics. Traditional methods are based on the processing of the hard-to-get Earth information. This information is usually so scarce that, by itself, it does not lead to any meaningful results; to come to useful conclusions (e.g., where oil most likely is), we must, in addition to the raw measurement results, use the experts' intuition and knowledge. In such situation, conclusions are reasonably *subjective*, and therefore, there is no question of estimating the accuracy of these conclusions: if the expert intuition turn out to be wrong (and once in a while it is wrong), the results are way off.

Space measurements radically change the situation. From the traditional geophysical situation where measurements results are scarce and hard-to-get, we get into a new situation (typical for space-related research) where space observations literally *flood* us with data, to the extent that we are unable to process it in real time (this inability is one of the main reasons why the PACES Center was established).

With this abundance of data, the results of data processing become more and more reliable, and it is reasonable to start asking the question: how accurate are they? This question is not easy to answer by traditional statistical methods, because different pieces of sensor information come from different sources, with different (and often unknown) error distribution. To estimate uncertainty of the results of data processing in such situations, we have combined statistical and interval methods; the resulting estimates are described in [15, 16].

Can the geophysical results be applied to other planets? Space analysis of Earth geophysical structures is not only helpful for geophysics, it also creates a testing ground for different methods that will later be applied to the research of the distant planets.

With this application in mind, it is important to clearly distinguish between the geophysical features that are specific to our Earth, and the features that are of fundamental origins and will, therefore, by typical for other planets as well.

The first question is: which planet areas are most informative? According to modern geophysics, the most interesting dynamical processes occur at the area where different tectonic plates interact. On Earth, in addition to heads-on *collisions* and *pull-apart* motions, there are few areas where plates collide at *oblique* angles.

On Earth, these oblique collisions are rare but important. Since these areas are rare on Earth, a question may be asked: will we find such areas on other planets? should we, therefore, prepare methods and models for handling these areas? Or should we rather concentrate on the methods of analyzing hands-on and pull-apart collisions?

In [17], fundamental geometric and topological methods are used to show that *oblique collisions are inevitable* on every planet on which surface is subdivided into tectonic plates, and therefore, their analysis is *important for future planetary missions*.

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