Interval-Valued Fuzzy Control in Space Exploration

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***Abstract*— This paper is a short overview of our NASA-supported research into the possibility of using interval-based intelligent control techniques for space exploration.**

Interval-values fuzzy sets were introduced by L. Zadeh, J. A. Goguen, and especially by I. B. Türkşen; they were actively used in expert systems by L. Kohout. Before we proceed to explain *how* to use them in fuzzy control, let us first explain *why* we need to use them.

I. Why intervals? Reasons Ia–c: Intervals naturally appear

Ia. Traditional fuzzy control techniques start with the expert's degree of belief that are represented by *numbers* from the interval $[0, 1]$.

- This use of numbers may be natural when we describe physical quantities, for which there exists a true value that can be, in principle, measured with greater and greater accuracy.
- However, for degrees of belief, numbers may not be the most adequate representation.

Indeed, how are the existing *knowledge elicitation* techniques determine these numbers?

- One of the possible techniques is to ask an expert to estimate his or her degree of belief by a number on a scale, say, from 0 to 10. Then, when an expert estimates this degree of belief by choosing, say, 6, we take $6/10 = 0.6$ as the numerical expression of the expert's degree of belief.
  At first glance, this may sound like a reasonable assignment, but in reality, the fact that an expert has chosen 6 does not necessarily mean that the expert's degree of belief is *exactly* equal to 0.6; it rather means that this degree of belief is closer to 0.6 than to the other values between which we have asked the expert to choose (i.e., to $0, 0.1, \ldots, 0.5, 0.7, \ldots, 0.9, 1.0$). Mathematically, values that are the closest to 0.6 form an interval $[0.55, 0.65]$. In other words, the only thing that we can conclude based on this choice is that the expert's *true* degree of belief belongs to the interval $[0.55, 0.65]$.
  In principle, we could try to get a more precise value of the degree of belief by asking an expert for a value on, say, a scale from 0 to 100, but hardly anyone can distinguish between degree of belief that correspond to, say, 63 and 64 on this scale. Thus, the interval $[0.55, 0.65]$ is the best we can get.
- Another way of determining the degree of belief is to *poll* experts. If 6 experts out of 10 believe that, say, a given value of x is small, then we take $6/10 = 0.6$ as the degree of belief $\mu_{\text{small}}(x)$ that this value x is small.
  Polls have their own margins of uncertainty. Hence, from a poll, we cannot extract the *exact* ratio of experts who believe that x is small; we can, at best, find an *interval* of possible values of this ratio.
  In principle, to get a narrower interval, we can ask more and more experts, but in reality, the number of experts is often limited, and asking all of them is not practically possible. As a result, the interval of possible values is the best we can get.

Similar conclusions can be obtained for all other methods of eliciting the values. For all these methods, an *interval* is a much more adequate description of the expert's degree of belief.

Ib. Even if we manage to get narrow enough intervals for degrees of belief, so that these original degrees of belief can be adequately described as numbers, in the fuzzy control methodology, we need to *process* these numbers. The first processing consists of applying *and* and *or* operations.

These operations, in their turn, must also be elicited from an expert so that they would be most adequate in describing what the experts mean when they use the corresponding connectives. We have already seen that eliciting *numbers* leads, in real-

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ity, to intervals. The resulting uncertainty is even worse if we try to elicit not a *single* number, but *several* different numbers that describe the desired *functions* $f_{\&}(a, b)$ and $f_{\vee}(a, b)$. As a result, instead of a *single* pair of functions, we, most probably, will get an *interval* of possible functions. If we apply this *interval* function to numerical input values, we get an *interval* of possible results.

Thus, even if we managed to avoid intervals on the first stage, they will appear on the second stage of fuzzy control methodology: when we combine the original degrees of belief into degrees of belief of different rules.

IC. Even if we fix *and* and *or* operations, for the same query, we can have *different* representations in terms of "and", "or", and "not". These different representations are equivalent in classical logic, but in fuzzy logic, they are not. As a result, depending on which representation we use, we may get different numerical answers to the query. Hence, if we only know the query itself, and we are not sure what "translation" into basic logical operations is the best, it is natural to return not a *single* numerical value, but the entire *interval* of possible values of degrees of belief that correspond to different possible translations.

In [61], we show how to compute this interval for different queries.

Comments.

- These three arguments (also given in [15, 23, 39, 50, 52]) do not exhaust all arguments in favor of intervals as a better way to describe uncertainty. Other arguments showing that two numbers represent uncertainty better are given, e.g., in [46].
- Intervals are very natural not only in the contents of fuzzy control, but also in computing in general. It suffices to say that actually, the modern calculus started with interval computations [7], and that intervals are the only sets whose use preserves the invertibility of arithmetic operations [22].
- So far, we have only said that intervals are a *more adequate* tool for describing expert knowledge. This, in itself, does not necessarily mean that fuzzy control that comes from using intervals is in any sense *better*. However, it is reasonable to expect that more adequate description of expert's knowledge leads to the fuzzy control that more adequately describes expert's high-quality control and is, therefore, of a better quality itself. In the next section, we will show that these expectations were correct: *interval control is*, indeed, in many cases *better*.

II. Why intervals? Reason II: Intervals lead to a better control

Interval-valued fuzzy control. If we use *intervals* of possible values of initial degrees of belief, then, on all further stages of fuzzy control methodology, we also have to use *intervals* of possible values.

General idea behind using intervals in fuzzy control ([39, 47])**.** The main idea of using an *interval* to describe the expert's degree of belief, instead of more traditional technique of picking *a* number from this interval, is that the *actual* (unknown) degree of belief is *guaranteed* to belong to the interval, but it may be *different* from the picked value.

From this idea, one can (informally) conclude that the resulting interval control is often better than the original number-valued control. It turned out that this indeed true [39, 46]:

Intervals lead to a more stable control. Traditional fuzzy control techniques, if used appropriately, lead to a control that is stable.

- However, if we use only a *single* picked value from the interval of possible values, we will get a control that is *stable for this* particular *value*, and may *not* be *stable* at all for the *actual* value.
- The only way to guarantee that the control is stable for the *actual* (unknown) value is to guarantee that it is stable for *all* values from this interval. This requires, at least, that the algorithm that computes the control values should have this interval at its disposal.

Thus, to improve stability, we must have an algorithm that processes *intervals* of degrees of belief (rather than picked numerical values).

Intervals lead to a smoother control. A picked value of degree of belief is, in general, unpredictably ("randomly") different from the actual value. As a result, the control $\bar{u}$ coming from the picked values, will "wobble" around the control that correspond to the actual (unknown) degrees of belief. The random wobbling around a smooth process usually makes it less smooth.

Thus, the way to avoid this wobbling (and to make control smoother) is to take into consideration that the actual values are within the *intervals*, and then, to choose the *smoothest* possible control within these intervals.

Intervals lead to a more robust control. The control that takes into consideration the possibility of slightly different inputs is, by definition, *more robust* than the control that is based only on the original picked values.

Intervals sometimes lead to a computation-

ally faster control. In general, computational simplicity is not the strongest point of interval computations (see below); however, in some situations, intervals do make computations faster.

Indeed, if we only have *numbers*, without any indication how accurate these numbers are, then, in order to guarantee the accuracy the resulting computations, we have to do all the data processing with all the digits of all these numbers.

If we know *intervals* instead of numbers, this means, in essence, that we know the *accuracy* of the input values. If the input values are known, say, with accuracy of 10%, then there is no much sense to have computations with much better accuracy, so we can use *fewer digits* in our computations and thus, make these *computations* much *faster*.

III. How to elicit interval membership functions?

In order to apply fuzzy control methodology, we must first elicit the membership functions and "and" and "or" operations that best describe the expert (or experts) whose opinions we are formalizing.

For this elicitation, we can use two sources of information:

- First, we can *interview* experts and try to extract the required information from their answers. For interval-valued degrees of belief, the corresponding problem is formulated and partially solved in [5].
- Interview is an ideal method, but often, experts who are very good in controlling are not that good in the ability to describe their control in words. Actually, the very necessity of fuzzy control comes from the fact that experts are not very good in describing their control strategies. For such experts, an important source of membership functions and other information is their actual control: we can
  - simulate different situations;
  - record how these experts would control the desired object, and then
  - try to extract their membership functions and other information from these records.

  In [4], we show that it is always possible to extract this information from the records, and we describe how exactly this can be done.

IV. The main problem of interval approach — computational complexity — and possible solutions of this problem

The problem. We have mentioned that the use of intervals often *improves the quality of* intelligent *control*. However, an apparent *disadvantage* of their use is that when we consider interval-valued instead of more traditional number-valued degrees of belief, we need to process twice as many numbers and therefore, the computational complexity (and thus, the computation time) increases. This increase can be very drastic: e.g.,

- While the solution of a linear systems $\sum a_{ij}x_j = b_i$ with crisp coefficients a_{ij} and b_i is a relatively easy problem, the solution of a linear system of equations with *interval* coefficients is, in general, computationally intractable (NP-hard) (see, e.g., [13, 33, 45]), even if we restrict ourselves to narrow intervals only [T3]. This computational complexity can be "explained" if we look at the geometric shape of the corresponding solution sets:
  - for crisp linear systems, the solution set is a *convex polytope*;
  - for interval linear systems with symmetric matrices $a_{ij} = a_{ji}$, the shape of the solution set becomes *piecewise-quadratic* [2];
  - for interval linear systems with dependent coefficients, we can have *arbitrarily complicated algebraic shapes* [1];
- Even when we have *explicit* computations (e.g., if we compute the value of a polynomial $f(x_1, \ldots, x_n)$) instead of solving systems of equations, for *interval-valued* inputs $x_1, \ldots, x_n$, the problem becomes NP-hard, and the shapes become algebraic shapes of arbitrary complexity [24, 32]. (In [29], a similar result is expressed in a slightly different form: if we want interval computations without roundoff errors, then we have to use algebraic numbers of arbitrary complexity.)
- For *expert systems* that use numerical degrees of belief, as soon as we have been able to express a given query as a logical combination of the statements from the system, computing the degree of belief in this query becomes a pretty straightforward and easy task. However, when we have *interval-valued* degrees of belief, the problem becomes NP-hard [T1].
- Closer to home, the problem of *eliciting* interval-valued *membership functions* is, in general, NP-hard [5].
- Also, the problem of finding the *optimal control* is, in general, NP-hard [57].
- Unfortunately, these results stay even if we consider a more realistic fuzzy-based formalization of feasibility [51, 52].

A general survey of such problems was given in [34] (see also [49]).

Comment. A similar trade-off between the control quality and its computational complexity can be observed if we compare *interval* methods with more traditional *statistical* methods:

- *interval methods* lead to *better estimates* [60], but
- *interval methods* are, in general, *more computationally complicated* [21].

How can we solve this problem? There are several possible ways to solve this problem:

– If we cannot find fast algorithms that work well in *all* cases, then we can look for algorithms that work well in *almost all* cases. In particular, for narrow intervals, the existence of such algorithms was shown in [44].

– If we cannot find an algorithm that works well in *almost all* cases, then, at least, we can try to look for specific cases in which fast interval algorithms are possible. In particular, we discovered such algorithms for the following problems that correspond to different stages of fuzzy control methodology:

- for some "and" and "or" operations, the problem of *eliciting* the interval-valued membership functions becomes computationally feasible [5];
- fast algorithms are also known for the case when the functions are *monotonic* [18]; *"and" and "or" operations* are usually monotonic;
- computing the range of *fractionally linear* functions [47]; this is important for applying *defuzzification*, which is usually described by a fractionally linear transformation;
- "smoothing" an interval function [43]; this is very important for designing a *smooth* control;
- locating local extrema of a function of one variable from interval measurement results [48, 59]; this is extremely important for *optimization*;
- finally, a fast algorithm is designed that checks *stability* of the resulting control [52].

– If we cannot find methods that are *guaranteed* to work well, then at least we may find *heuristic* methods that may often work. As part of this research, we have proposed and analyzed both the modifications of the existing heuristic methods, such as *genetic algorithms* [14] and *chemical computing* [28, T2], and proposed new interval-based heuristics [28, 58].

For heuristic methods, two questions naturally arise:

- We know that sometimes these methods do not lead to the best possible results. Can we, given the inputs, check whether this method will work or not? and how good the results are?
- In many cases, heuristic methods contain several parameters that need to be tuned. Depending on how we choose the values of these parameters, we may get very good results or very lousy results. How can we choose the optimal values of these parameters?

In this research, we attack both question:

- In [38], we design a method for estimating the quality of interval computations. To be more precise, there exist several methods that compute the *enclosure* (superset) of the desired interval. Methods from [38] generate a *subset* of this interval. If the resulting two interval are close, this means that the enclosure is a good estimate of the desired interval.
  In a more general context, the *rating* of different methods is proposed and justified in [T4].
- To find the optimal values of the parameters of heuristic methods, we use the general group-theoretic (symmetry) approach [42]. In particular, in [53], we show that re-scaling, a useful heuristic technique in fuzzy control and in genetic algorithms, should be best avoided in the case of complete uncertainty.

– If the interval-related *mathematical* problem that we are trying to solve is still too complicated, we may want to check whether this mathematical problem is indeed an adequate formalization of the original real-life problem.

In many cases, as Zadeh himself mentions, the complexity of the model is caused by the fact that the model tries to describe the original *low-granularity* problem, with few distinct levels of a certain quantity (like "small", "medium", and "large") by a model in which this quantity is described by a real number and thus, has infinitely possible values (*high granularity*).

Discovering that this indeed is the source of the problem is one thing; the next important step is to see what we can do in this situation to speed up computations. In [31], we describe how we can possibly do computations *directly* with low-granularity values, without translating them into high-granularity numbers.

– Finally, if we *cannot* think of any way of making an *algorithm faster*, we can still speed up the computations if we make *interval operations hardware supported* (and thus, faster).

It is impossible to hardware support all possible operations with intervals. In view of that, in [40, 54], we analyze (and solve) the problem of choosing the interval operations whose hardware support will lead to the largest computation speed-up; the answer, crudely speaking, is as follows: in addition to interval analogs of standard arithmetic operations, we must support an operation of weighted dot (scalar) product $a_1, \ldots, a_n, b_1, \ldots, b_n \to \sum w_i \cdot a_i \cdot b_i$.

V. Applications to Space-Related Data Processing

A. Data Processing is Important

Computation of the optimal control strategy is not the only space-related computation. Indeed, why do we need to launch space missions in the first place? One of the main objectives of the space flights is to bring the *information* about objects and processes, both in space and on the Earth. This information rarely comes in the desired form, it usually requires some *processing*.

Computation of the best control strategy can also be viewed as a *particular case* of data processing: namely, we take as inputs the sensor data, and we return the desired control. It is therefore reasonable to try to apply the methods and results, that were originally designed for control-related data processing, to general space-related data processing.

B. Major Areas of Space-Related Data Processing

In order to describe how these ideas can be used in space-related data processing, let us first enumerate the major areas of space-related data processing:

- At present, most space missions occur in the close vicinity of the Earth, and all of them are in the Solar system. Thus, the major area of space-related data processing is the analysis of near-Earth environment from the results of data processing.
- The near-Earth environment is not the only area about which we learn more after the space missions. Space is also the area from where, undisturbed by the Earth atmosphere, we can:
  - clearly observe the distant bodies and thus, get a large-scale picture of our Galaxy and of the Universe as a whole;
  - precisely trace the effects of the gravitation and thus, get a very clear picture of the relativistic effects and, in general, of the space-time (in particular, Dr. Jorge Lopez from Physics Department of the University of Texas at El Paso is doing this data processing from JPL);
  - observe high-energy particles and processes and thus, get a clearer understanding of the fundamental physical processes.
- Last but not the least, space flights, especially near-Earth space flights, brings us a lot of *geophysical* information, i.e., information about our Earth. The importance of this application area is emphasized by the fact that the Mission to Planet Earth is one the main missions of NASA.

C. Near-Earth observations

For near-Earth observations, we can formulate the following three problems:

- First, we would like to *estimate* the *accuracy* of the existing indirect measuring techniques.
- Second, for the situations when the resulting accuracy is not sufficient, we would like to design *new*, more accurate indirect measurement (= data processing) methods.
- Traditional data processing results in *numbers* that still have to be analyzed. Therefore, it is desired, in addition to this traditional data processing, to have more *intelligent* data processing that would provide us directly with the answers to the *fundamental* questions about the Solar system, questions that we are really interested in.

As part of the project, we solved the simplest cases of all these three problems:

Error estimation.

- Most of the instruments and sensors used in space missions are *similar to the instruments used on Earth*, and so, we can use the results of error estimation obtained in the analysis of Earth measurements (see, e.g., [3, 6, 12, 15, 23, 37]).
- There are, however, a few instruments and sensors that are *more specific* for space environment.

Namely, one of the main advantages of space observations is that in space, there is practically no atmosphere, and therefore, *optical observations* can be drastically more accurate than on Earth. This comment relates both:

- to *passive* observations, when we simply use an orbital telescope to observe the light coming from the celestial bodies, and
- to *active* observations, when we artificially "brighten" the objects and then observe the reflected light.

To "brighten" the images, we must use a very strong source of light; so far, the strongest sources of light are *lasers*, so, we arrive at the problem of *estimating accuracy for laser observations.*

A particular case of this problem was considered and solved in [56].

New data processing methods. As we have just mentioned, most near-Earth measurements are very similar to Earth measurements. There are, however, a few things that are radically different in space. The major difference is that:

- on Earth, all the matter is usually in one of the three main states: solid, liquid, and gas.
- In space, many substances are in the fourth state: of *plasma*, where, instead of electrically neutral atoms, we have *charged* particles: electrons and ions.

The abundance of charged particles often creates currents, magnetic fields, etc., that are much stronger than we are used to, and therefore, cannot be directly measured by means of traditional sensors. For these measurements, we need a new methodology.

In [41], we design a new method of measuring string current by measuring magnetic fields that these currents generate; so far, the algorithm is applied to the Earth situations in which strong current are artificially created: to the string currents used in aluminum production.

Fundamental questions about the solar system. So far, the Solar systems works as a clockwork; it looks like catastrophes are highly unprobable. However, the huge masses of celestial bodies, together with the high speeds, make every collision truly catastrophic. So, one of the most fundamental questions is: *Is the Solar system truly stable or a big collision is inevitable*?

This problem is very *difficult to solve numerically* because small numerical uncertainties (that are inevitable in calculations) increase exponentially and make the results of long-term numerical calculations useless for predictions. So, the only way to guarantee stability is to have predictions with *a guaranteed accuracy*.

In [30], we apply interval methods to the stability problem: namely, we show that within a certain reasonable hypothesis, our *Solar system is stable*.

D. Relativistic Effects and the Structure of Space-Time

For these applications, to measuring geometry of space-time, we have two types of results:

- First, we show that the corresponding problems are, in general, very *computationally complicated* [19]. Even when the corresponding problems are computationally feasible [20], the problems of measuring proper distances and proper times in space-time geometry are much more complicated than the problems of measuring distances in Euclidean space [25].
- Second, we show that the general *group-theoretic* methodology can be successfully applied to these problems.

  In particular, we show that reasonable axioms of space-time geometry that are usually formulated in geometric and causal terms can be reformulated in terms of symmetries [27]. This general reformulation turns out to be quite useful: e.g., causality explains the previously unexplained physical fact about symmetries: that spatial and temporal translations commute [26].

E. Fundamental Physical Processes

Modern physics is based on *quantum mechanics*, which is usually interpreted in *probabilistic* terms. At first glance, there seems to be no big need for using *fuzzy* and/or *interval* methods. However, a more attentive analysis reveals some fundamental problems in traditional probabilistic approach:

- First, the equations of quantum filed theory often lead to meaningless *infinities* instead of the physically meaningful finite values. There exist several semi-heuristic methods of handling these infinities, but it is definitely desirable to avoid them from the very beginning.
- For some possible physical processes that are seriously considered in modern physics (e.g., for acausal processes), the standard probability approach encounter problems (see, e.g., [8]).

To handle both problems, we first showed, in [17], that *standard quantum mechanics* approach can be viewed as a *particular case* of the more general *fuzzy* approach (of which interval uncertainty is another particular case), and that many supposedly specifically quantum phenomena can be thus explained [36] as pure mathematical consequences of the formalism rather than a necessity for a new specifically quantum approach. With this embedding, we have a more general formalism, and we show that both problems can be naturally handled within this more general formalism:

- In [16], we show that if we take into consideration *measurement uncertainty*, in particular, *interval uncertainty*, then the equations of physics become consistent.
- In [8], we show that the natural description of *acausal* processes leads to *non-probabilistic uncertainty.*

As a side effect of these results, in [35], we *explain* why the group-theoretic (*symmetry*) approach, an approach which has originated on physics and which we have so successfully used in our research, is useful in physics.

F. Mission to Planet Earth

Specific feature of geophysical data processing is that we also have lots of Earth data. The processing of geophysical data is one of the main areas of space-related data processing. In particular, this data processing is one of the main areas of the NASA Pan-American Center for Earth and Environmental Studies (PACES) that operates in El Paso, Texas.

The specific feature of this application area (as opposed to pure space research) is that, in addition to information coming from space flights, there is also *lots* of geophysical *information* about the same areas coming from the *Earth measurements*. It is therefore important to process *both* types of measurement results.

With new space data, a new problem arises: estimating accuracy of the results of data processing. Many data processing methods have been developed in traditional geophysics. Traditional methods are based on the processing of the hard-to-get Earth information. This information is usually so scarce that, by itself, it does not lead to any meaningful results; to come to useful conclusions (e.g., where oil most likely is), we must, in addition to the raw measurement results, use the experts' intuition and knowledge. In such situation, conclusions are reasonably *subjective*, and therefore, there is no question of estimating the accuracy of these conclusions: if the expert intuition turn out to be wrong (and once in a while it is wrong), the results are way off.

Space measurements radically change the situation. From the traditional geophysical situation where measurements results are scarce and hard-to-get, we get into a new situation (typical for space-related research) where space observations literally *flood* us with data, to the extent that we are unable to process it in real time (this inability is one of the main reasons why the PACES Center was established).

With this abundance of data, the results of data processing become more and more reliable, and it is reasonable to start asking the question: how accurate are they? This question is not easy to answer by traditional statistical methods, because different pieces of sensor information come from different sources, with different (and often unknown) error distribution. To estimate uncertainty of the results of data processing in such situations, we have combined statistical and interval methods; the resulting estimates are described in [9, 10].

Can the geophysical results be applied to other planets? Space analysis of Earth geophysical structures is not only helpful for geophysics, it also creates a testing ground for different methods that will later be applied to the research of the distant planets.

With this application in mind, it is important to clearly distinguish between the geophysical features that are specific to our Earth, and the features that are of fundamental origins and will, therefore, by typical for other planets as well.

The first question is: which planet areas are most informative? According to modern geophysics, the most interesting dynamical processes occur at the area where different tectonic plates interact. On Earth, in addition to heads-on *collisions* and *pull-apart* motions, there are few areas where plates collide at *oblique* angles.

On Earth, these oblique collisions are rare but important. Since these areas are rare on Earth, a question may be asked: will we find such areas on other planets? should we, therefore, prepare methods and models for handling these areas? Or should we rather concentrate on the methods of analyzing hands-on and pull-apart collisions?

In [11], fundamental geometric and topological methods are used to show that *oblique collisions are inevitable* on every planet on which surface is subdivided into tectonic plates, and therefore, their analysis is *important for future planetary missions.*

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