Non-Equilibrium Thermodynamics Explains Semiotic Shapes: Applications to Astronomy and to Non-Destructive Testing of Aerospace Systems

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ABSTRACT

Celestial bodies such as galaxies, stellar clusters, planetary systems, etc., have different geometric shapes (e.g., galaxies can be spiral or circular, etc.). Usually, complicated physical theories are used to explain these shapes; for example, several dozen different theories explain why many galaxies are of spiral shape. Some rare shapes are still difficult to explain.

It turns out that to explain these "astroshapes", we do not need to know the details of *physical* equations: practically all the shapes of celestial bodies can be explained by simple *geometric* invariance properties. This fact explains, e.g., why so many different physical theories lead to the same spiral galaxy shapes.

This same physical idea is used to solve a different problem: the optimal sensor placement for non-destructive testing of aerospace systems.

KEYWORDS: *astrogeometry, semiotics, symmetry groups, non-equilibrium thermodynamics, non-destructive testing, aerospace structures*

1. SEMIOTIC SHAPES IN ASTRONOMY: FORMULATION OF THE FIRST PROBLEM

From the computer viewpoint, an astronomical image is a *set of pixels* of different brightness. However, astronomers traditionally interpret these images in terms of certain geometric *shapes*, usually, described in semiotic terms (by *words* and *symbols*). For example, they talk about spiral or elliptical galaxies, etc. This language is very productive, because it enables astronomers to predict new results. However, the very origin of these shapes remains somewhat a mystery.

To be more precise, there are several dozens theories that explain, e.g., the spiral galaxy shape (see, e.g., [2,9,10]), but the very fact that there are so many different theories for explaining the same observations probably means that the physical *details* involved in these theories *are not needed*, and these shapes can follow from *fundamental principles*.

In this paper, we show that that this is indeed the case: *we can explain these shapes by using the fundamental physical ideas of symmetry and non-equilibrium thermodynamics.*

2. MAIN PHYSICAL IDEA

The initial state of the Universe was highly symmetric. To find out how shapes have been formed, let us start from the beginning of the Universe (for a detailed physical description, see, e.g., Zeldovich and Novikov [13]). The only evidence about the earliest stages of the Universe is the cosmic 3K background radiation. This radiation is highly homogeneous and isotropic; this means that initially, the distribution of matter in the Universe was highly homogeneous and isotropic. In mathematical terms, the initial distribution of matter was invariant w.r.t. arbitrary shifts and rotations.

We can also say that the *initial distribution was invariant* w.r.t. dilations if in addition to dilation in space (i.e., to changing the unit of length), we accordingly change the unit of mass.

In the following text, we will denote the corresponding transformation group (generated by arbitrary shifts $\vec{x} \to \vec{x} + \vec{a}$, rotations, and dilation $\vec{x} \to \lambda \cdot \vec{x}$) by G.

Dynamic equations are also symmetric. On the astronomical scale, of all fundamental forces (strong, weak, etc.) only two forces are non-negligible: *gravity* and *electromagnetism.* The equations that describe these two forces are invariant w.r.t. arbitrary shifts, rotations, and dilations in space. In other words, these interactions are *invariant* w.r.t. our group G.

The problem: our world should be symmetric, but it is not.

- The *initial distribution* was *invariant* w.r.t. G;
- the *evolution equations are* also *invariant*;

hence, we will get $G-$invariant distribution of matter for all moments of time. But our world is not homogeneous. Why?

Solution: spontaneous symmetry violation. The reason why do not see this homogeneous distribution is that this highly symmetric distribution is known to be *unstable*: If, due to a small perturbation, at some point $\vec{a}$ in space, density becomes *higher* than in the neighboring points, then this point $\vec{a}$ will start *attracting* matter from other points. As a result, its density will increase even more, while the density of the surrounding areas will decrease. So, arbitrarily small perturbations cause drastic changes in the matter distribution: matter concentrates in some areas, and shapes are formed. In physics, such symmetry violation is called *spontaneous.*

Non-equilibrium thermodynamics explains why perturbations usually preserve some symmetry. What kind of perturbations are possible? In principle, it is possible to have a perturbation that changes the initial highly symmetric state into a state with no symmetries at all, but statistical physics teaches us that it is much more probable to have a gradual symmetry violation: first, some of the symmetries are violated, while some still remain; then, some other symmetries are violated, etc. (Similarly, a (highly organized) solid body normally goes through a (somewhat organized) liquid phase before it reaches a (completely disorganized) gas phase.) At the end, we get the only stable shape: *rotating ellipsoid.*

This idea leads ot an explanation of all possible astroshapes. Before we reach the ultimate ellipsoid stage, perturbations are invariant w.r.t. some subgroup G' of the initial group G. If a certain perturbation concentrates matter, among other points, at some point $\vec{a}$, then, due to invariance, for every transformation $g \in G'$, we will observe a similar concentration at the point $g(\vec{a})$. Therefore, the shape of the resulting concentration contains, with every point $\vec{a}$, the entire *orbit* $G'(\vec{a}) = \{g(\vec{a}) \,|\, g \in G'\}$ of the group G'. Hence, the resulting *shape consists of* one or several *orbits* of a group G'.

3. THE RESULT OF PHYSICAL ANALYSIS: DESCRIPTION OF ASTROSHAPES

In view of the above analysis, to describe all possible *shapes* of celestial bodies, it is sufficient to describe all possible *orbits* of subgroups G' of the group G (= all shifts, rotations, and dilations). In this paper, we will show that this description really describes all known astroshapes. (Some of these results were first announced in [3–5,6–8].)

A word of warning: geometric shapes are only approximate. Objects of nature can only *approximately* be described by geometric figures. Correspondingly, in our physical explanation, perturbations are only *approximately* invariant w.r.t. G'. The farther away from the point $\vec{a}$, the less similar is the point $g(\vec{a})$ to the point $\vec{a}$. Therefore, in reality, we may observe not the *entire* orbit, but only a *part* of it.

Possible orbits. $0-$, $1-$, and $2-$dimensional orbits of continuous subgroups G' of the group G are easy to describe:

0: The only $0-$dimensional orbit is a *point.*

1: A generic $1-$dimensional orbit is a *conic spiral* that is described (in cylindrical coordinates) by the equations $z = k\rho$ and $\rho = R_0 \exp(c\varphi)$. Its limit cases are:

- a *logarithmic* (Archimedean) *spiral*: a planar curve ($z = 0$) that is described (in polar coordinates) by the equation $\rho = R_0 \exp(c\varphi)$.
- a *cylindrical spiral*, that is described (in appropriate coordinates) by the equations $z = k\varphi$, $\rho = R_0$.
- a *circle* ($z = 0$, $\rho = R_0$);
- a *semi-line* (*ray*);
- a *straight line.*

2: Possible 2-D orbits include:

- a *plane*;
- a *semi-plane*;
- a *sphere*;
- a *semi-plane*;
- a *circular cylinder*, and
- a *logarithmic cylinder*, i.e., a cylinder based on a logarithmic spiral.

Possible orbits are exactly possible shapes. Comparing these orbits (and ellipsoids, the ultimate stable shapes) with astroshapes enumerated in Vorontsov-Veliaminov [12], we conclude that:

- First, our scheme describes all observed connected shapes.
- Second, all above orbits, except the logarithmic cylinder, have actually been observed as shapes of celestial bodies.

For example, according to Chapter III of Vorontsov-Veliaminov [12], galaxies consist of components of the following geometric shapes:

- *bars* (cylinders);
- *disks* (parts of the plane);
- *rings* (circles);
- *arcs* (parts of circles and lines);
- *radial rays*;
- *logarithmic spirals*;
- *spheres*, and
- *ellipsoids*.

The only orbit-originated shape that is not in this list is logarithmic spiral. It is easy to explain why logarithmic cylinder was never observed: from whatever point we view it, the logarithmic cylinder blocks all the sky, so it does not lead to any visible shape in the sky at all. With this explanation, we can conclude that we have a *perfect explanation of all observed astroshapes.*

Comment: we can also explain difficult-to-explain disconnected shapes. In the above description, we only considered connected *continuous* subgroups $G' \subseteq G$. Connected continuous subgroups explain connected shapes.
It is natural to consider disconnected (in particular, discrete) subgroups as well; the orbits of these subgroups leads to disconnected shapes. Thus, we can explain these shapes, most of which modern astrophysics finds pathological and difficult to explain (see, e.g., Vorontsov-Veliaminov [12], Section I.3).
For example, an orbit O of a discrete subgroup G'' of the 1-D group G' (whose orbit is a logarithmic spiral) consists of points whose distances r_n to the center forms a geometric progression: $r_n = r_0 \cdot k^n$. Such dependence (called Titzius-Bode law) has indeed been observed (as early as the 18th century) for planets of the Solar system and for the satellites of the planets (this law actually led to the prediction and discovery of what is now called asteroids). Thus, we get a *purely geometric explanation of the Titzius-Bode law.*

Less known examples of disconnected shapes that can be explained in this manner include:

- several parallel equidistant lines (Vorontsov-Veliaminov [12], Section I.3);
- several circles located on the same cone, whose distances from the cone's vertex form a geometric progression (Vorontsov-Veliaminov [12], Section III.9);
- equidistant points on a straight line (Vorontsov-Veliaminov [12], Sections VII.3 and IX.3);
- "piecewise circles": equidistant points on a circle; an example is MCG 0-9-15 (Vorontsov-Veliaminov [12], Section VII.3);
- "piecewise spirals": points on a logarithmic spiral whose distances from a center form a geometric progression; some galaxies of Sc type are like that (Vorontsov-Veliaminov [12]).

Not only shapes can be this explained. This idea also explains relative *frequency* of different shapes, the directions of *rotation* and *magnetic* field, possible *evolution* of geometric shapes, etc. (see, e.g., [5]).

4. ALTERNATIVE EXPLANATION: OPTIMIZATION UNDER UNCERTAINTY AND CORRESPONDING OPTIMAL SHAPES

There is an alternative way of analyzing the shapes, that does not refer to physics at all, but is instead looking for the best *approximations* of (unknown) actual shapes.
If we use this idea, we face the problem of selecting *the best* family of images for use in extrapolation under an *uncertain* optimzality criterion. How can we solve this problem? It turns out that for *every* optimality criterion that satisfies the natural symmetry conditions (crudely speaking, that the relative quality of two image reconstructions should not change if we simply shift or rotate two images), the extrapolation shapes that are *optimal* with respect to this criterion can be described as *orbits* of the subgroups of the corresponding symmetry group.
As a result, we get exactly the shapes used in astronomy (such as spirals, planes, spheres, etc.) The details of this description are given in [3,4].

5. OPTIMAL SENSOR PLACEMENT FOR NON-DESTRUCTIVE TESTING OF AEROSPACE SYSTEMS: THE SECOND PROBLEM

Testing is extremely important. Structural integrity is extremely important for airplanes, because in flight, the airframe is subjected to such stressful conditions that even a relatively small crack can be disastrous. This problem becomes more and more important as the aircraft fleet ages.

Sensors must be placed. At present, most airplanes do not have built-in sensors for structural integrity, and even those that have such sensors, do not have a sufficient number of them, so additional sensors must be placed to test the structural integrity of a airframe.
Each integrity violation (crack etc.) starts with a small disturbance that is only detectable in stressful in-flight conditions. Therefore, to detect these violations as early as possible, we should complement on-earth testing by in-flight measurements.

Optimal sensor placement: a problem. Sensors attached outside the airframe interfere with the airplane's well-designed aerodynamics; therefore, we should use as few sensors as possible. The problem is, *given the number of sensors that we can locate on a certain surface of an airframe, what are the optimal placements of these sensors, i.e., locations that allow us to detect the locations of the faults with the best possible accuracy.*
For *future* aircraft, we have a similar problem of sensor placement. The ideal design of a future airplane should include built-in sensors that are pre-blended in the perfect aerodynamic shape. Each built-in sensor is expensive to blend in and requires continuous maintenance and data processing, so again, we would like to use as few sensors as possible.

This optimality problem is difficult to formulate in precise terms. Both for aging and for the future aircraft, the ideal formulation of the corresponding optimization problem is to *minimize* the average detection error for fault locations. However:

- this ideal formulation requires that we *know the probabilities* of different fault locations and the probabilities of different aircraft exploitation regimes.
- In reality, especially for a new aircraft, we *do not have that statistics*, and for the aging aircraft, the statistics gathered from its earlier usage may not be applicable to its current state.

Therefore, instead of a *well-defined* optimization problem, we face a *not so well defined* problem of optimization under uncertainty. Since the problem is not well defined, we cannot simply use standard numerical optimization techniques, we must use intelligent techniques.

Geometric approach. The problem of choosing an optimal sensor placement can be formulated in *geometric* terms: we need to *select points* (sensor placements) on a *surface* of the given structure.
To solve this problem, we use the experience of solving similar symmetry-based geometric problems of optimization under uncertainty in image processing and image extrapolation (see above). Since the basic surface shapes are *symmetric*, a similar symmetry-based approach can be applied to the problem of optimal sensor placement. For the simplest surfaces such as planes, cones, etc., this general approach describes several geometric patterns that every sensor placements which is optimal with respect to reasonable (symmetric) optimality criterion must follow.

The use of neural networks. We then use *neural networks*:

- first, to confirm that these placement patterns indeed lead to better fault location, and
- second, to select a pattern that leads to the best results for each particular problem.

Discussion about the results. The resulting placements are different for different problems: For example,

- when we test on-earth, then our main goal is not to miss the crack; as long as we detected it, we can always perform additional measurements to determine its location with any desired accuracy.
- In flight, however, detecting the crack is not enough; in a fly-by-wire aircraft, we may need to adjust the control algorithm so as not to stress the faulty surface. For that, we need to know where exactly this fault is located.

Space structures: a similar problem. A similar problem of optimal placement of sensors for non-destructive testing can be formulated and solved for space structures.

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