INTERVAL APPROACH TO NON-DESTRUCTIVE TESTING OF AEROSPACE STRUCTURES AND TO MAMMOGRAPHY

Keith Worden
Dynamics Research Group, Department of Mechanical Engineering
University of Sheffield, Sheffield S1 3JD, United Kingdom

Roberto Osegueda, Carlos Ferregut, Soheil Nazarian, Eulalio Rodriguez, Debra L. George, Mary J. George, and **Vladik Kreinovich**
FAST Center, University of Texas at El Paso
El Paso, TX 79968, USA, email vladik@cs.utep.edu

Olga Kosheleva and **Sergio Cabrera**
Department of Electrical and Computer Engineering
University of Texas at El Paso, El Paso, TX 79968, USA
email {olga,cabrera}@ece.utep.edu

Aerospace Testing: Why

One of the most important characteristics of the plane is its weight: every pound shaved off the plane means a pound added to the carrying ability of this plane. As a result, planes are made as light as possible, with its "skin" as thin as possible. However, the thinner the layer, the more vulnerable is the resulting structure to stresses and faults, and a flight is a very stressful experience. Therefore, even minor faults in the plane's structure, if undetected, can be disastrous. To avoid possible catastrophic consequences, before the flight, we must thoroughly check the structural integrity of the plane.

Aerospace Testing: How

Some faults, like cracks, holes, etc., are external, and can, therefore, be detected during the visual inspection. However, to detect internal faults (cracks, holes, etc.), we must somehow scan the inside of the thin plate that forms the skin of the plane. This skin is not transparent to light or to other electromagnetic radiation; very energetic radiation, e.g., X-rays or gamma-rays, can go through the metal, but it is difficult to use on such a huge object as a modern plane.

The one thing that easily penetrates the skin is vibration. Therefore, we can use sound, ultrasound, etc., to detect the faults. Usually, a wave easily glosses over obstacles whose size is smaller than its wavelength. Therefore, since we want to detect the smallest possible faults, we must choose the sound waves with the smallest possible wavelength, i.e., the largest possible frequency. This frequency is usually higher than the frequencies that we hear, so it corresponds to *ultrasound*.

Ultrasonic scans are indeed one of the main nondestructive NDE tools; see, e.g, [2, 3, 4, 5, 8].

Aerospace Integrity Testing is Very Time-Consuming and Expensive

One possibility is to have a point-by-point ultrasound testing, the so called *S-scan*. This testing detects the exact locations and shapes of all the faults. Its main drawback, however, is that since we need to cover every point, we get a very time-consuming (and therefore, very expensive) testing process.

A faster idea is to send waves through the material so that with each measurement, we will, be able to test not just a single point, but the entire line between the transmitter and the receiver. To make this procedure work, we need special signals called *Lamb waves*.

There are other testing techniques. All these techniques aim at determining whether there is a fault, and if there are faults, what is the location and the size of each fault.

How Can We Save Time and Money?

In spite of many time-saving ideas, for each of these methods, we must still scan a huge area for potential small faults. As a result, testing requires lots of time, and is very expensive. How can we save the time and cost of testing? Our main idea is this:

The existing testing procedures are very expensive and time-consuming because they try not only to check whether there is a fault, but also to find its location and size. If our only goal is to detect the fault, and we are not interested in its exact location, then the problem becomes much simpler and hopefully, easier to solve. Therefore, we suggest to make a two-step testing:

- First, we apply a simpler test to check whether there is a fault.
- Only when the first test detects the presence of a fault, we run more expensive tests to locate and size this fault.

This two-step procedure is very similar to medical testing: In medical testing, first, the basic parameters are tested such as body temperature, blood pressure, pulse, etc. If everything is OK, then the person is considered healthy. Only if something is not OK, then the whole battery of often expensive and time-consuming tests is used to detect what exactly is wrong with the patient.

So the question is: How can we detect the presence of a fault?

Our Main Idea

Let us first describe this idea in general terms. For testing, we send a signal and measure the resulting signal. The input signal can be described by its intensity $x_1, \ldots, x_n$ at different moments of time. The intensities $y_1, \ldots, y_m$ of the resulting signal depend on x_i: $y_j = f_j(x_1, \ldots, x_n)$, where the functions f_j depend on the tested structure.

Usually, we do not know the exact analytical expression for the dependency f_j, so we can use the fact that an arbitrary continuous function can be approximated by a polynomial (of a sufficiently large order). Thus, we can take a structure, try a general linear dependency first, then, if necessary, general quadratic, etc., until we find the dependency that fits the desired data.

If a structure has no faults, then the surface is usually smooth. As a result, the dependency f_j is also smooth; we can expand it in Taylor series. Since we are sending relatively weak signals x_i (strong signals can damage the plane), we can neglect quadratic terms and only consider linear terms in these series; thus, the dependency will be *linear*.

A fault is, usually, a violation of smoothness (e.g., a crack). Thus, if there is a fault, the structure stops being smooth; hence, the function f_j stops being smooth, and therefore, linear terms are no longer sufficient. Thus, *in the absence of fault, the dependence is linear, but with the faults, the dependence is non-linear.*

So, we can detect the fault by checking whether the dependency between y_j and x_i is linear.

Comment. The idea that non-linear terms can be helpful has been suggested some time ago, see, e.g., [2].

Non-Linearity Experimentally Confirmed

The first experimental confirmation that faults do cause non-linear terms was presented in [9]; it was shown that for an ultrasonic scan:

- for a fault-less plate, the dependence between the transmitted signal $x(t)$ and the measured signal $y(t)$ is linear i.e.,

  $$y(t) = \int A(t-s) \cdot x(s)\, ds$$

  for some function $A(t)$;
- for a plate with a fault, this dependence is non-linear: namely, cubic terms must be taken into consideration.

Moreover, our preliminary analysis of data from [9] has shown that the amplitude of the cubic terms is roughly proportional to the cube of the linear fault size. Thus, not only the non-linear terms indicate the *presence* of the fault, but also the value of the cubic term can be used to determine the *size* of the fault.

In [9], the *pseudo-random* signals $x(t)$ were used. These signals are good to analyze all possible dependencies, but they are reasonably difficult to generate.

To simplify the experiments, in our new tests, we use *harmonic* signals $x(t) = A \cdot \cos(\omega \cdot t)$. For these input signals, we also detected non-linearity: namely, while the transmitted signal only has components of the original frequency ω, the measured signal also had a Fourier component corresponding to the triple frequency 3ω. This component is 3 to 5 times above noise, so it indicates that we are observing non-linear effects.

How can we explain these non-linear effects?

Mechanical Explanation of Non-Linearity

In this section, we present a simplified mechanical explanation of non-linearity. This explanation is too oversimplified to explain the *quantitative* experimental results, but it explains, on the *qualitative* level, why non-linearities do occur.

In order to understand how non-linear effects can occur, let us first describe how the signal travels through a fault-less plate. In this case, at the location of the transmitter, we send, at any given moment of time t, the signal $x(t) = A \cdot \cos(\omega \cdot t)$.

This signal travels to the receiver (measuring device) with a velocity equal to the speed of sound. For simplicity, we can assume that the plate is homogeneous, so at any point, we have the same speed of sound v. Thus, while traveling from the transmitter to the receiver, the signal gets delayed by the amount of time $\Delta t = d/v$, where d is the distance between the transmitter and the receiver. As a result, at any moment of time t, the values of the observed signal $y(t)$ is proportional to value $x(t-\Delta t)$ that the input signal had Δt seconds ago: $y(t) = k \cdot x(t-\Delta t)$, where the coefficient k describes the loss of amplitude.

Thus, for a fault-less plate, we indeed have a linear dependence between the transmitted signal $x(t)$ and the measured signal $y(t)$.

Let us now consider the case when a fault lies between the transmitter and the receiver. This fault may be a crack or a hole. In this case, we can also use the formula $y(t) = k \cdot x(t - \Delta t)$, where Δt is the delay. However, this delay can no longer be computed simply as d/v, because, in addition to going straight through the material, the signal has to go either through or around the crack. In both cases, the presence of the crack changes the travel time:

- If the ultrasound has to travel through air, then it is delayed because the speed of sound in the air is smaller than the speed of sound in the solid body.
- If the ultrasound has to go around the crack, then the speed of sound stays the same, but the length of the path increases, and so the signal is also delayed.

In both cases, the delay Δt between the transmitter and the receiver can be computed as $\Delta t = d/v + k_f \cdot d_0$, where d_0 is the linear size of the fault, i.e., the distance between the front and the rear borders ("walls") of the fault area (front and rear with respect to the transmitter), and the coefficient k_f describes how fast the signal passes the fault area. As a result, the measured signal is equal to $y(t) = k \cdot x(t - \Delta t) = k \cdot A \cdot \cos(\omega \cdot t - \omega \cdot \Delta t)$. Since we are interested in detecting small faults, the value d_0 is small, so we can expand the expression for $y(t)$ in terms of d_0 and keep only the first few terms. As a result, we get the following formula

$$y(t) = A \cdot \cos(\omega \cdot t - \omega \cdot d_f/v) +$$

$$k_f \cdot d_0 \cdot A \cdot \sin(\omega \cdot t\omega \cdot d_f/v) + o(d_0). \qquad (1)$$

Before we send the signal, the plate is immobile, and the distance d_0 stays constant: $d_0(t) = d_0^{(0)}$. However, as we transmit the signal $x(t)$, the plate starts vibrating, and this vibration changes the position of both borders and therefore, changes the distance d_0: $d_0 = d_0(t)$. In order to describe this change, let us denote the distance between the transmitter and the fault's front border by d_f. By the time the signal reaches this left border, it is delayed by the time d_f/v, i.e., takes the form $x_{\text{front}}(t) = k_{\text{front}} \cdot A \cdot \cos(\omega \cdot t - \omega \cdot d_f/v)$. This vibration causes the corresponding change in the location of this front border: instead of being equal exactly to d_f, this location oscillates around x_f. At any given moment of time, the change in location is proportional to the amplitude $x_{\text{front}}(t)$ of oscillating signal:

$$d_{\text{front}}(t) = d_f + k_{\text{mov}} \cdot x_{\text{front}}(t) = d_f +$$

$$k_{\text{mov}} \cdot k_{\text{front}} \cdot A \cdot \cos(\omega \cdot t - \omega \cdot (d_f/v)),$$

for some coefficient k_{mov}.

Similarly, the signal that passes to the rear border gets delayed by $\approx d_f/v + k_f \cdot d_0^{(0)}$. Thus, the location location of the rare border also changes, as

$$d_{\text{rear}}(t) = d_f + k_{\text{mov}} \cdot x_{\text{rear}}(t) = d_f +$$

$$k_{\text{mov}} \cdot k_{\text{front}} \cdot A \cdot \cos(\omega \cdot t - \omega \cdot (d_f/v) - \omega \cdot k_f \cdot d_0^{(0)}).$$

As a result of these slightly different oscillations, the size $d_0(t) = d_{\text{real}}(t) - d_{\text{front}}(t)$ also changes with time. We have already mentioned that the size d_0 is small, so we can expand the expression for $d_0(t)$ in terms of $d_0^{(0)}$ and keep only the first few terms. As a result, we get the following formula:

$$d_0(t) = d_0^{(0)} +$$

$$k_{\text{mov}} \cdot k_{\text{front}} \cdot A \cdot \omega \cdot k_f \cdot d_0^{(0)} \cdot \sin(\omega \cdot t - \omega \cdot (d_f/v))$$

$$+ o(d_0^{(0)}). \qquad (2)$$

Substituting (2) into (1), we get, in $y(t)$, in addition to terms proportional to $\cos(\omega t)$, also quadratic terms $\sin^2(\omega t)$ which lead to double frequency terms in the Fourier transform of $y(t)$. These terms are proportional to A^2.

Similarly, we get cubic terms, etc.

Why These Non-Linear Terms are Important

When we add a fault to the ultrasound-scanned plate, two changes occur:

- first, linear terms change;
- second, non-linear terms appear.

Since the fault is small, the change is linear terms is much higher than the non-linear terms.

Thus, if we *know* what the plate was supposed to be like and how it reacted to scanning before the fault appeared, it is much easier to detect the presence of the fault by observing the changes in the linear response than by measuring its non-linear component.

However, in some cases (e.g., in mammography, see below) we *do not know* the linear response of the fault-less object. Or, alternatively, we may know the initial response, but we also know that this response can change not only because of faults, but also because of stress, material wariness, and other factors that do not necessarily mean that there is a dangerous fault inside. In such cases:

- we *cannot* detect the fault by comparing the current *linear* response with the ideal one;
- however, if we detect *non-linear* terms, it is a clear indication that there are some faults inside.

How Can We Use Non-Linear Terms to Detect a Fault: Idealized Case of a Purely Harmonic Signal

Let us first consider an idealized case when the transmitter sends a pure harmonic signal

$$x(t) = A \cdot \cos(\omega \cdot t)$$

(with a frequency ω that corresponds to a Lamb wave). For such a signal, a general time-invariant cubic dependence leads to a signal

$$y(t) = y_0 + y_1 \cdot \cos(\omega \cdot t + \varphi_1) +$$

$$y_2 \cdot \cos(2\omega \cdot t + \varphi_2) + y_3 \cdot \cos(3\omega \cdot t + \varphi_2),$$

where

$$y_i = C_{i0} + C_{i1} \cdot A + C_{i2} \cdot A^2 + C_{i3} \cdot A^3.$$

In the linear case, the only non-zero terms are C_{00} and C_{10} (that describe the noise) and C_{01} and C_{11}. All other terms describe non-linear dependence of $y(t)$ on $x(t)$ and thus, indicate the presence of faults.

Thus, for this idealized case, to detect a fault, we can do the following:

- For the same frequency ω, send a harmonic signal with this frequency ω several times, with different amplitudes $A^{(k)}$, i.e., send the signals $x^{(k)}(t) = A^{(k)} \cdot \cos(\omega \cdot t)$ for several different values $A^{(k)}$, $1 \le k \le K$, where $K \ge 2$.
- For each of these experiments, we measure the output signal $y^{(k)}(t)$, and compute the Fourier components $f_i^{(k)}$ corresponding to $i \cdot \omega$, $i = 0, 1, 2, 3$.
- Then, we use, e.g., the least squares method to estimate the desired coefficients C_{ij} from the equations

$$f_i^{(k)} = C_{i0} + C_{i1} \cdot A^{(k)} + C_{i2} \cdot (A^{(k)})^2 +$$

$$C_{i3} \cdot (A^{(k)})^3.$$

If one of the non-linear terms is reliably different from 0, this means there is a fault.

How Can We Use Non-Linear Terms to Detect a Fault: A More Realistic Approach

In reality, the transmitter does not send a *harmonic* signal, it sends a *narrow-band* signal instead, i.e., a signal that covers all frequencies from some interval $[\omega^-, \omega^+]$. As a result, we must modify the above algorithm as follows: as $f_i^{(k)}$, we take not the Fourier component corresponding to $i \cdot \omega$, but the *total amplitude* of all Fourier components from the interval $[i \cdot \omega^-, i \cdot \omega^+]$.

Interval Methods are Needed

After K measurements, we have K sets of data $\tilde{x}_1^{(k)}, \ldots, \tilde{x}_n^{(k)}, \tilde{y}_1^{(k)}, \ldots, \tilde{y}_m^{(k)}$, $1 \le k \le K$. Often, we do not know the probabilities of different measurement errors, we only know the upper bounds for these errors. So, we know the *intervals* $X_1^{(k)}, \ldots, X_n^{(k)}, Y_1^{(k)}, \ldots, Y_m^{(k)}$ of possible values of the measured quantities. We want to check whether this dependence can be linear, i.e., whether these exist coefficients c_{ij} for which, for every k and j, $\sum c_{ij} \cdot x_j^{(k)} \in Y_j^{(k)}$ for some $x_i^{(k)} \in X_i^{(k)}$. This is a known problem of interval computations: check whether the given system of interval linear equations is solvable (here, the unknowns are c_{ij}, interval coefficients are $X_i^{(k)}$ and $Y_j^{(k)}$).

Our main concern is not to miss the fault, so we need *guaranteed* methods. Thus, we need to use interval (guaranteed) methods for solving linear interval systems (see, e.g., [1, 6, 7]).

Applications to Mammography

The main problem of mammography is to detect small non-smoothnesses in the mammal (small clots, cracks, etc.), which may indicate a tumor. When formulated in these terms, the problem sounds very similar to the problem of aerospace testing: in both cases, we must detect possible non-smoothness.

Thus, we can use the above idea in mammography as well: if the dependence is linear, everything is OK, else further testing is needed.

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